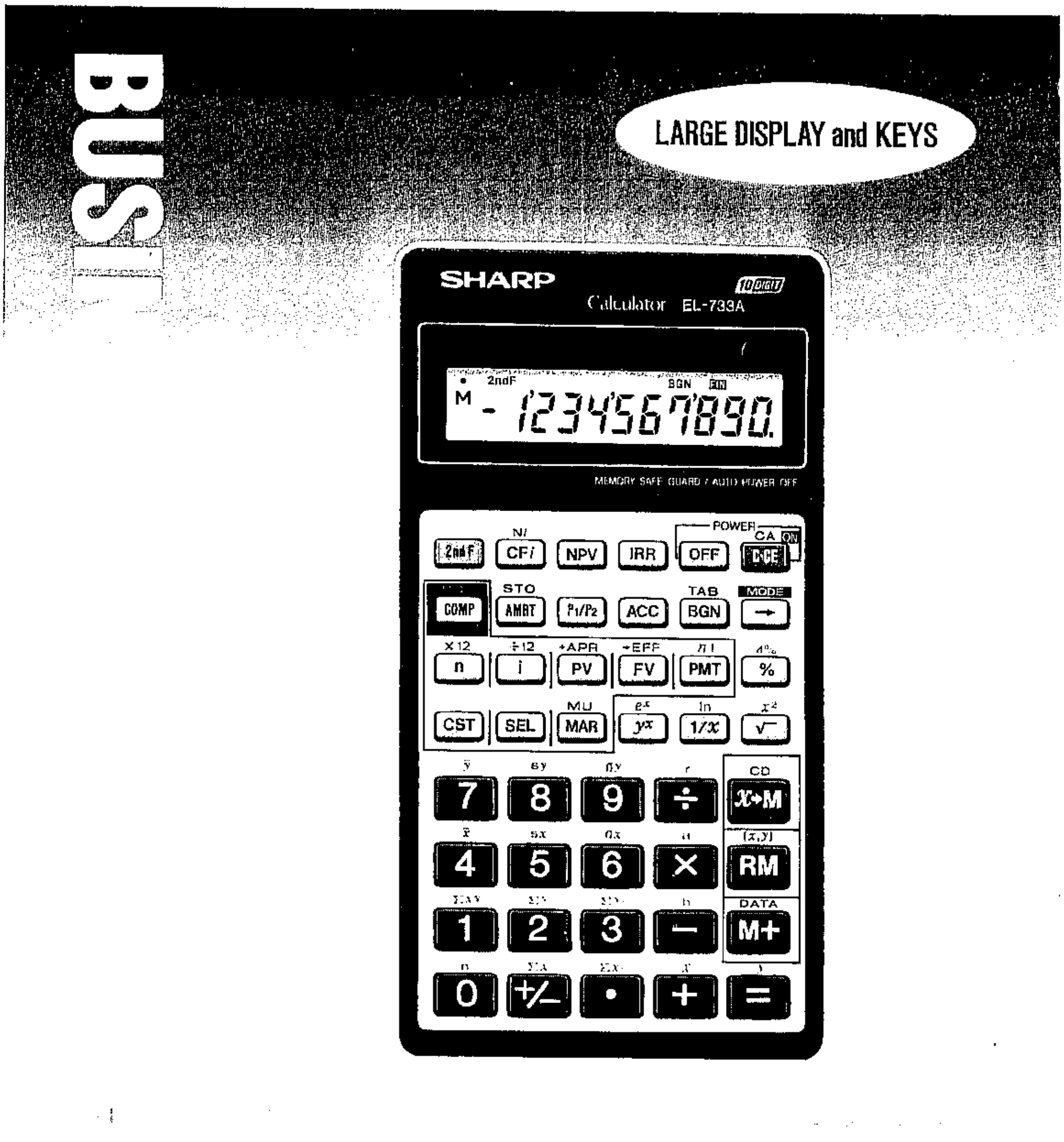
SHARP
EL-733A
FINANCIAL CALCULATOR
OWNER'S MANUAL AND SOLUTIONS HANDBOOK



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The Sharp EL-733A Financial Calculator

Owner's Manual And Solutions Handbook

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Congratulations!

By choosing the SHARP EL-733A Financial Calculator, you passed an important initial test in financial decision-making: you chose the calculator that gives you the most financial computing power for the money. This computing power, along with the built-in high quality that is a tradition at Sharp Electronics, will make your EL-733A a long-time friend and business partner.

How To Use This Manual

The SHARP EL-733A Owner's Manual And Solutions Handbook has been designed for two purposes. It is both a reference book and a fairly complete course on financial calculation.

The complete table of contents that begins on the next page, along with the function index and alphabetical index in the back of this manual, make it perfect for quick reference. When you are in a hurry and need a quick reminder to solve a particular problem, one of the indexes should lead you directly to the appropriate page or pages.

Ideally though, we recommend that you take an evening or so sometime in the near future and read the majority of this book. The emphasis of this book is on financial problem solving, as the title implies. The chapter on financial calculations contains pictorial techniques that can considerably simplify the ways that you approach financial problems and investment analyses. You will find that the time you spend with this book is an investment in itself, and that investment in time will continue to bring you rich returns.

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Chapter 1. Getting Started

This first chapter covers the basics of using your SHARP EL-733A. Keying in numbers and correcting errors, arithmetic calculations, percentage calculations including the [CST], [SEL], [MAR], and [MU] keys, and other general math functions that include [y^x], [1/x], [√], [e^x], [ln], [n!], and [x^2] are all part of Getting Started. This chapter contains important background information, and it is a good chapter to read to get comfortable with using the EL-733A.

The Keys Of The Calculator

Look at the keys on your EL-733A. Every key has a function printed on it, plus some keys have functions printed above them. The function on the face of a key is called the primary function and the function printed above a key is called the second function (frequently abbreviated as 2nd F). Some keys have no second function, and thus, nothing is printed above them.

Turning The Calculator On And Off

To turn on the calculator, press the red [C·CE] key. When the calculator is off, this key serves as the ON key. Notice that ON is actually the third function of the [C·CE] key. Its primary function is C-CE (Clear or Clear Entry), its second function (printed above it) is CA (Clear All), and off to the side the characters [ON] are printed. This is the only key on the calculator with three functions associated with it, and the [ON] function only comes into play when the calculator is off.

The [OFF] key is used to turn the calculator off. The EL-733A will turn itself off if no keys are pressed for several minutes. This is the AUTO POWER OFF feature, and it is included in the calculator to protect the batteries.

The EL-733A Display

When the calculator is on, the display contains at least a number and a good battery indicator. The good battery indicator is a dot in the upper left corner of the display. When this dot no longer appears in the display, your batteries need to be replaced as soon as possible. Battery replacement is covered in the back of this manual.

Other indicators can also appear in the display. For example, the letter M will appear in the left side of the display if some number other than zero is stored in the calculator's memory register. The other indicators that can appear in the display will be discussed later, in their appropriate sections.

Keying In Numbers

To key a number into the display, simply press the keys that represent the digits of that number. For example, to key in the number 1.226, press [1] [•] [2] [2] [6].

[+/-]: Changing The Sign Of A Number

To change a number from positive to negative or vice versa, press the [+/-] key. If you press [+/-] now the number 1.226 in your display will change to –1.226. Press [+/-] again, and it will change back to 1.226. The [+/-] key is used frequently in financial calculations.

[2ndF] [TAB]: Adjusting The Displayed Decimal Places

The [2ndF] [TAB] key is used to adjust the number of displayed decimal places. From the above example, press [=] to tell the EL-733A that you are done keying in a number (1.226), then press [2ndF] [TAB] [3]. The calculator displays all three decimal places. Press [2ndF] [TAB] [2] to display 1.23.

The EL-733A rounds the number in the display to the specified display format, however this rounding only takes place in the display. The number it stores and uses in its calculations contains every digit (up to 10 digits). By having the rounding occur only in the display, the EL-733A minimizes rounding errors throughout your calculations.

Pressing [2ndF] [TAB] [•] sets the calculator to display every digit after the decimal point, except for trailing zeros. With this display setting, if you key in 4.250000000 [=] the display will show 4.25, whereas if you key in 4.250000001 [=] you will see 4.250000001. Trailing zeros are not shown.

Digit Grouping

Digits in numbers over one-thousand are grouped using an apostrophe in the display rather than a comma. For example, key in the number 1'234'567'890.

The reason that an apostrophe is used where one may expect a comma is that the EL-733A is sold to an international audience. The majority of the world prefers that, for example, the number one-million be written as 1.000.000,00 (with a comma as the radix), whereas people in the U. S. prefer 1,000,000.00. So, to avoid confusion, SHARP chose a format that is easy for everyone to understand.

Very Large Numbers (Scientific Notation)

Few people ever get to work with money amounts that exceed 9'999'999'999.00, but, if you are one of those people whose assets exceed ten billion, you may be interested in how the EL-733A handles large numbers like these.

Any number greater than or equal to ten billion is represented in scientific notation. For example, the number fifteen billion would be displayed as 1.5000 10, which means 1.5 times ten to the tenth power (1.5×10^{10}).

To key in large numbers in scientific notation, key in the power of ten first by using the [y^x] key, then multiply by the mantissa (the other part). For example, to key in fifteen billion, press 10 [y^x] 10 [×] 1.5 [=].

Clearing The Display And Correcting Errors

Two keys are provided for correcting errors in the display. The [→] key allows you to backspace incorrect digits from the number you are currently keying in, and the [C•CE] key clears the number in the display. As an example, key in the number 6.22895, then use [→] to change the 8 to a 5. Once you have 6.22595 in the display, press [C•CE] to clear the display back to zero.

Each time you press the [C•CE] key, you clear one number from your calculation. For example, if you key in 5 + 6 then press [C•CE], the 6 is cleared and the 5 reappears in the display. Pressing [C•CE] again clears the 5 taking you back to where you started.

The second function of the [C•CE] key is the clearing function [2ndF] [CA] (Clear All). This function clears all the registers associated with a particular mode, preserving only the M register.

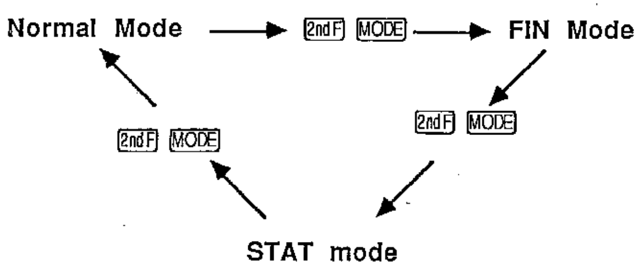
Occasionally, you may press a key that does not make sense to the calculator (for example, try to divide by zero). An "E" will appear in the lower left corner of the display indicating that an error occurred. To clear this error and continue working, press [C•CE].

Three Calculation Modes

Your SHARP EL-733A has three calculation modes: NORMAL mode, FIN mode (financial mode), and STAT mode (statistics mode).

The mode that your calculator is in dictates which functions are immediately available. When you are working a financial problem, you most likely will want the EL-733A to be in FIN mode. Likewise, statistics problems generally require the calculator to be in STAT mode.

To change modes, simply press the keys [2ndF] [MODE] ([MODE] is the second function of the [→] key, thus to access that function you need to first press the [2ndF] key). Press these two keys repeatedly and you will notice that the display tells you which mode you are in. In FIN mode, a box containing "FIN" appears in the display. In STAT mode, a box containing "STAT" appears, and in NORMAL mode, no box appears.



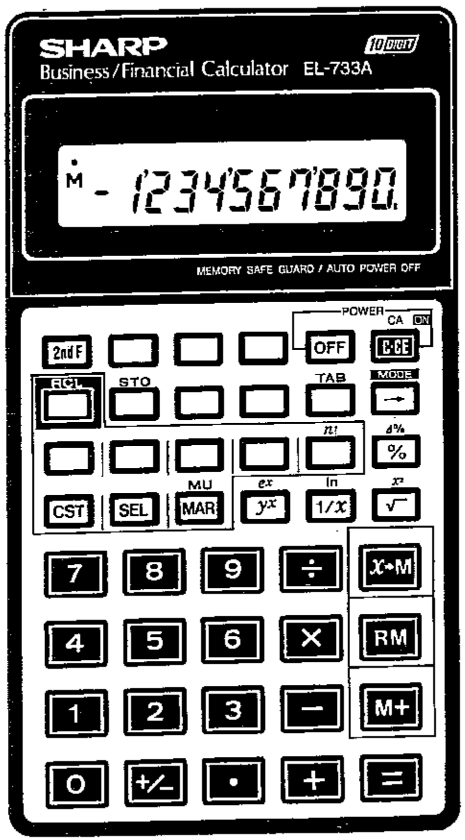
FIN mode and STAT mode are the subjects of chapters 2 and 3 respectively. The remainder of this chapter covers NORMAL mode. Before you continue, press the keys [2ndF] [MODE] until no mode box appears in the display, indicating that you are in NORMAL mode.

The [2ndF] Key

If you press [2ndF] once, an indicator "2ndF" comes on in the display. When this indicator is on, keys with second functions take on those meanings (but only if those second functions are active functions in the mode that you are in). Pressing [2ndF] again takes the "2ndF" indicator out of the display and restores the primary meanings to the keys.

Active Functions In NORMAL Mode

When your calculator is in NORMAL mode (neither the FIN or STAT indicator appears in the display), only certain functions are active. The following diagram shows you the active functions:



The blank keys on the previous diagram represent inactive functions. If you press a key that is inactive, nothing happens.

Arithmetic In NORMAL Mode

Most people have had at least some experience with using the four functions [+], [−], [×], and [÷]. Arithmetic problems are simply keyed in as you would say them. And, you can chain several calculations together as shown in the examples below. All of the arithmetic functions are completed in the order that you key them in.

For the following examples, set your calculator to display four decimal places before you start (press [2ndF] [TAB] [4]).

Example: Calculate 7.75 + 1.25.

Solution: 7.75 [+] 1.25 [=] Result: 9.0000

Example: Calculate (993.7 + 688.3) ÷ 21.0

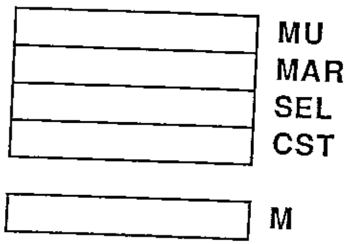
Solution: 993.7 [+] 688.3 [÷] 21 [=] Result: 80.0952

Example: Calculate 8 x (2265 − 1104)

Solution: 2265 [−] 1104 [×] 8 [=] Result: 9288.0000

Memory Registers In NORMAL Mode

The EL-733A has five memory registers available for use in NORMAL Mode. These memory registers are called M, CST, SEL, MAR, and MU. **Another 25 registers are available for use in FIN mode**, but those registers are described starting on page 32. Picture the NORMAL mode registers as boxes that can each hold one number:



As long as you stay in NORMAL mode, numbers stored in any of these' registers are preserved, even when the calculator is turned off. The registers CST, SEL, MAR, and MU are described starting on page 25. The M register is described below.

THE M REGISTER

The three keys that help you use the M register are the [X-M], [RM], and [M+] keys. The M register can be used to help get through certain arithmetic problems in NORMAL mode, to temporarily store results, or, because the number in the M register is preserved when the calculator is turned off, you can store an important number there for an extended period of time.

The M register is active in both NORMAL and FIN modes. It is not active in STAT mode.

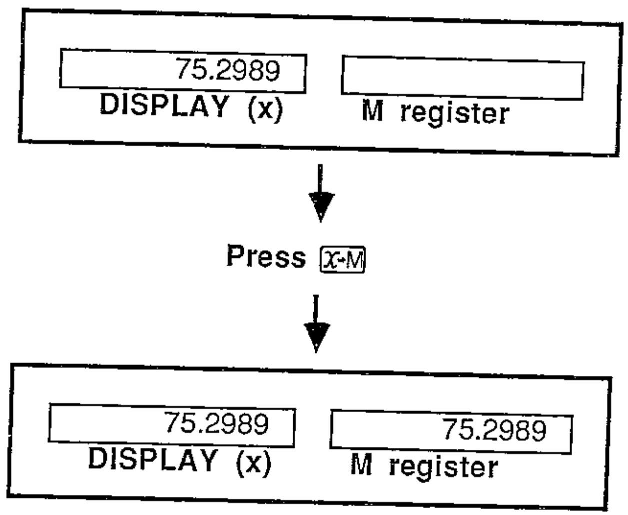
STORING A NUMBER IN THE M REGISTER

To store a number in the M register, simply press [X-M]. The new number will replace any number that was stored there. When some number other than zero is stored in the M register, an "M" indicator comes on in the upper left corner of the display.

Example: Store the number 75.2989 in the M register.

Solution: 75.2989 [X-M]

The following diagram shows you what happens when you press [X-M].



Notice that the "M" indicator is now on in the upper left corner of the display. This indicator is on whenever a number other than zero is stored in the M register.

RECALLING THE NUMBER IN THE M REGISTER

Turn the calculator OFF. Then turn it back on. The display is cleared when you turn it on, but notice that the "M" memory indicator is still on in the display. Press [RM] to recall the number in memory. The number 75.2989 will appear in the display.

After you press [RM], the "M" indicator in the display stays on to tell you that 75.2989 is still stored in the M register.

USING A STORED NUMBER IN A CALCULATION

To use the number stored in M in one of your calculations, simply press [RM] at the point in the calculation when you would otherwise have to key in that number. The following example demonstrates using the M register in your calculations.

Example: Calculate 75.2989 + 33.5727

Solution: [RM] [+] 33.5727 [=] Result: 108.8716

Example: Calculate 99.2115 – 75.2989

Solution: 99.2115 [–] [RM] [=] Result: 23.9126

Again, when you recall the number from the M register, it does not change that number. That stored number will not change until you change it by storing a new number in the M register or by adding or subtracting from that number as described in the following section.

USING [M+] TO SUM NUMBERS IN MEMORY

To add a number to the number in the M register, press the [M+] key. The sum of the displayed number and the number in the M register will be the new number in the M register.

Example: Add 25 to the 75.2989 in the M register.

Solution: 25 [M+] Result: 25.0000

Example: Subtract 0.2989 from the number in M.

Solution: .2989 [+/–] [M+] Result: –0.2989

Example: Recall the number in the M register.

Solution: [RM] Result: 100.0000

CLEARING MEMORY

To clear the M register, press [0] [X→M] or [RM] [+/–] [M+]. The "M" memory indicator in the display will disappear.

Examples In NORMAL Mode

The rest of this chapter contains useful examples of calculating using NORMAL mode. Work through those examples that apply to your current needs or that may apply to your future needs.

If you have an idea about how to start an example without looking at the keystroke solution, by all means try to come up with the result. Trying an example before looking at the solution tends to speed up the learning process, regardless of whether or not you obtain the correct result.

BALANCING YOUR CHECKBOOK

One calculation procedure that most people face monthly is the challenge of reconciling a bank's statement of balance on their checking account with their own register of that account. Some people view this process as a pain-in-the-neck and try to get around it one way or another. Others enjoy taking a few minutes to verify their wealth (or lack thereof) on a monthly basis.

Whether you fall into one of the above catagories or somewhere in between, you know that balancing the checkbook has not gotten any easier in recent years. The recent development of Automatic Teller Machines now allow you to withdraw cash directly from your checking account without having to write a check. The convenience is great, but it is often at the expense of accurate recordkeeping. Cash withdrawals can go unrecorded (especially when more than one person uses the same account), making the monthly statement/register reconciliation an even greater necessity.

The exact procedure for balancing the checkbook can take on many forms, and you may use a procedure that has been handed down in your family from generation to generation,

or you may use the form that most banks include with the statement. Whichever procedure you use, you will find that your SHARP EL-733A can be a big help.

We are about to describe a six point procedure for reconciling a bank statement with your check register. This probably isn't the procedure you use, but it may be interesting to see another approach. In any case, the way that we use the M register in the example that starts on page 20 will probably be useful to you in your own account balancing procedure.

This six point procedure assumes the following:

- That your bank statement provides a list of your checks in order by check number (most statements provide this).
- That a break in the sequence of check numbers in that ordered list of checks is indicated with an asterisk (or by some other means).
- That you keep a written register of the checks, deposits, and withdrawals on your account.

The six point procedure for reconciling the bank statement with your checking account register is as follows:

1. Note the statement date and the last check number that appears on the statement.
2. Draw a line in your register under the last check to appear on the statement, and compute the register balance to that line (if you haven't already). Start with that balance.
3. Add any deposits that appear below the line in your register and that occurred before the statement date (they appear on the statement).

4. Subtract any cash withdrawals that appear below the line in your register and that occurred before the statement date.

5. Mark any checks (with a √ or *) that appear above the line in your register that don't show on the statement (and that haven't shown on previous statements). The quickest way to find these outstanding checks is to note the breaks in sequence in the list of numbered checks on your statement. Add the amounts of these outstanding checks.

6. Add any interest earned on the account and subtract any service charge. Record interest earned and/or service charge in your register (after the last entry).

The number that you arrive at using the above procedure should exactly match the ending balance on the statement.

Take a look at the following example:

Example: You just received your latest bank statement dated Jun 1, 1989. The ending balance on the statement is $977.39 and the last check number on the statement is 1612. The calculated balance up to check number 1612 in your register is 907.93. And, one cash withdrawal on May 26, for 40.00, appears in your register after check 1612.

There are six checks missing from the sequence of numbered checks on the statement, but three of those occur at the beginning of the list, so you assume that they are not outstanding. The three outstanding checks are for the amounts 26.22, 65.00, and 11.75 (according to your register).

You earned $6.49 in interest on this statement. Does your register match the statement?

Solution: This calculation can be done either by using memory arithmetic or by using a chain calculation. The memory arithmetic solution is described first, then the chain calculation keystrokes are shown. The results are shown in [2ndF] [TAB] [2] format.

First store the balance from your check register (after check 1612) into M. Press 907.93 [X→M].

Subtract the $40 withdrawal that occurred after check 1612 but before the statement date. Press 40 [+/-] [M+].

Add the three outstanding checks. Press 26.22 [M+] 65 [M+] 11.75 [M+].

Add the interest you earned. Press 6.49 [M+]. And finally, recall the result by pressing [RM]. The result is $977.39, so your register matches the statement exactly. Good recordkeeping! Be sure to record in your register the interest that you earned.

If you don't want to use memory arithmetic, but would rather do a chain calculation, the keystrokes would be 907.93 [−] 40 [+] 26.22 [+] 65 [+] 11.75 [+] 6.49 [=]. This actually takes less keystrokes.

PERCENTAGE CALCULATIONS ([%] [Δ%])

Two percent calculation keys are provided on the EL-733A: [%] (percent) and [Δ%] (percentage difference). These functions are handy for calculating quick percentages and percentage increases and decreases. Look at a couple examples:

Example: What is 7% of 66?

Solution: 66 [×] 7 [%] Result: 4.62

Example: Calculate 24.99 + 6%.

Solution: 24.99 [+] 6 [%] Result: 26.49

Example: Decrease 24'000 by 16%.

Solution: 24'000 [−] 16 [%] Result: 20'160.00

Generally the [%] key is used in conjunction with the [+], [−], or [×], keys for calculations similar to the ones just shown. It can also be used in conjunction with the [÷] key where it is essentially the same as pressing [×] 100 [=].

Example: What is 6'599'975'227 ÷ 4%?

Solution: 6'599'975'227 [÷] 4 [%] Result: 1.65 11

If the notation of the result (scientific notation) is confusing, you may want to review page 8. This notation is used to represent very large numbers in the display. Also, if you don't understand the apostrophe that appears in the display, review page 8.

Notice that in the above solution, you can replace the [%] key with [×] 100 [=], and you will get the same result. As a business person, you may rarely use [%] with [÷].

The [Δ%] key is always used in conjuction with the [−] key. If you wish to know the percent increase or decrease between two numbers, start the calculation as if you are just taking the difference (subtracting), but instead of pressing the [=] key to complete the calculation, press [2ndF] [Δ%].

Example: Sales in your company were $75'000.00 during it's first year of operation. The second year sales were $116'000.00. Second year sales were what percent greater than first year sales?

Solution: 116'000 [−] 75'000 [2ndF] [Δ%] Result: 54.67

This result tells you that sales for the second year were up 54.67% over the sales of the first year. The percentage difference is calculated with the sales of the first year as the base for the percentage calculation.

Pressing the [Δ%] key, tells the calculator to perform the operation between the two numbers you just entered, to divide by the second number you entered, and then to multiply by 100. When the operation between the two numbers that you enter is [−], the result of pressing [2ndF] [Δ%] is the percentage increase or decrease between the two numbers (a decrease will be displayed as a negative). If you use [Δ%] in conjunction with the operations [+], [×], or [÷], it works through the formula described in the first sentence of this paragraph, but the meaning of the result is not as clearcut.

In summary again, to calculate a percent increase or decrease, start the calculation as if you were going to take the difference using [−], but instead of pressing [=], press [2ndF] [Δ%].

Example: As the owner of a trucking firm, you recently had wind deflectors installed on the tops of all your tractors in an effort to save on fuel costs. Your average mileage jumped from 5.7 miles per gallon to 6.2 miles per gallon. By what percentage did you cut your fuel costs?

Solution: Before installing the deflectors, fuel for each mile on the road cost you the price of a gallon of gas divided by 5.7. Now each mile only costs you the price of a gallon of gas divided by 6.2. These are the keystrokes:

6.2 [1/x] [−] 5.7 [1/x] [2ndF] [Δ%]   Result: –8.06

Installing the deflectors cut fuel costs by 8.06%.

MARK-UP AND MARGIN ([CST], [SEL], [MAR], AND [MU])

Besides the [%] and [Δ%] keys on your calculator for percentage calculations, you will find the very useful functions [CST], [SEL], [MAR], and [2ndF] [MU]. Each one of these keys represents a memory register in the calculator. You can store any number in these registers when you are in NORMAL mode. Each register can hold one number.

	MU
	MAR
	SEL
	CST

However the true utility of these keys is not just in the fact that you can store numbers in them for later use, but in the fact that the numbers in these registers have a special meaning to the calculator. The main function of these keys is to allow you to do some powerful business percentage calculations.

The meanings of these keys are as follows:

[CST] Cost
[SEL] Selling Price
[MAR] Margin
[MU] Mark-Up

Look at the following examples. Your calculator must be in NORMAL mode, and the display should be set to [2ndF] [TAB] [2] in order to get the exact results that we show:

Example: In your furniture business, you like to see about a 95% mark up from your cost to the retail price of each item. A certain couch costs you $455.70. What price do you sell it for?

Solution: 95 [2ndF] [MU] 455.70 [CST] [COMP] [SEL] Result: 888.62

As you work through the above solution, you store the mark-up in the MU register by pressing 95 [2ndF] [MU]. Then you store the cost in the CST register (455.70 [CST]) and compute the selling price by pressing [COMP] [SEL]. (The [COMP] key always means "compute."

Question: What is the margin on each of these couches that you sell?

Solution: After solving the above example, simply press [COMP] [MAR]. Result: 48.72

In the above examples you have learned two things:

1. To store a number in one of the four registers CST, SEL, MAR, and MU, all you need to do is key in the number and press the key that names the desired register.

2. To compute one of the four values CST, SEL, MAR, or MU, based on numbers that you have just stored, press [COMP] followed by the key that names the value you wish to compute.

Understanding the above two items is the beginning of learning to get the most out of the four functions [CST], [SEL],

[MAR], and [MU]. And this understanding about storing and computing also applies to the functions [n], [i], [PV], [FV], and [PMT] (five important functions available in FIN mode).

Example: In the same furniture business described in the last example, your cost on a 4' x 6' oriental rug is $502.40. Using the same mark-up and margin, what is the selling price

Solution: 502.40 [CST] [COMP] [SEL] Result: 979.68

Notice that once a number is stored in one of these four registers, it only changes if you store a new number there or if you compute a new value (or if you press [2ndF] [CA], which "clears all" registers except M).

After completing the last series of examples, the numbers in the four business calculation registers are as shown below:

95.00	**MU**
48.72	**MAR**
979.68	**SEL**
502.40	**CST**

To verify that these registers contain those numbers, you can use the [RCL] (Recall) function. Press [2ndF] [RCL] [CST] to see 502.40. Press [2ndF] [RCL] [SEL] to see 979.68, [2ndF] [RCL] [MAR] to see 48.72, and [2ndF] [RCL] [2ndF] [MU] to see 95.00.

You can use the numbers stored in these registers just like you have used the numbers stored in the M register (explained starting on page 14) in your calculations, except no register arithmetic key like the [M+] key is available.

MATH FUNCTIONS [y^x], [1/x], [ln], [e^x], [√], [n!], AND [x^2]

Seven math functions are available on the EL-733A and the use of these functions is fairly straightforward. These seven functions are available in all three calculating modes. All of the functions except for [y^x] operate instantaneously on the number showing in the display. With [y^x], you need to key in one number (y), press [y^x], key in another number (x), and press [=].

Some examples of using these math functions follow. To get the exact results that we show, your calculator must be in NORMAL mode with the display set to [2ndF] [TAB] [2].

Example: Calculate 1/9, 1/7, and 1/12.

Solution: 9 [1/x] Result: 0.11
7 [1/x] Result: 0.14
12 [1/x] Result: 0.08

Example: What is the square root of 289?

Solution: 289 [√] Result: 17.00

Example: What is 2^5 (two to the fifth power)?

Solution: 2 [y^x] 5 [=] Result: 32.00

Example: What is the cube root of 755?

Solution: The key to solving this type of problem (a root other than the square root) is to know that the nth root of a number is equal to that number raised to the 1/n power. In this case, you need to raise 755 to the 1/3 power in order to calculate the cube (or third) root.

755 [y^x] 3 [1/x] [=] Result: 9.11

To verify this result, you can raise it back to the third power by pressing [y^x] 3 [=]. This will give you 755.00.

Example: What is 275^2 (275 squared)?

Solution: 275 [2ndF] [x^2] Result: 75'625.00

Example: What is 6! (six factorial = 1 x 2 x 3 x 4 x 5 x 6)

Solution: 6 [2ndF] [n!] Result: 720.00

Example: What is $e^{0.226}$?

Solution: 0.226 [2ndF] [e^x] Result: 1.25

Example: What is the natural log of 422?

Solution: 422 [2ndF] [ln] Result: 6.05

The functions [ln] and [eˣ] are based on the number "e" which is approximately 2.718281828. An example of statistical regression using these functions is described starting on page 157.

ARITHMETIC WITH CONSTANTS

The EL-733A is equipped with a built-in constant feature that allows repetitive calculations. You can perform the same function with the same number without having to re-enter that number and function. Try the following keystrokes.

10 [+] 20 [=] Result: 30.0000

Now 20 is a constant for further additions:

60 [=] Result: 80.0000

Subtraction is similar:

100 [−] 25 [=] Result: 75.0000
40 [=] Result: 15.0000

With multiplication, the first number entered is the constant:

3 [×] 5 [=] Result: 15.0000
10 [=] Result: 30.0000

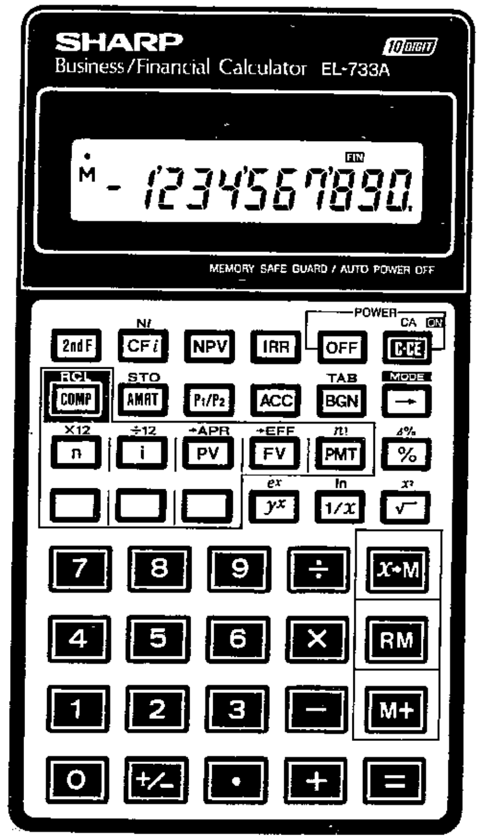
With division, the second number entered is the constant:

15 [÷] 3 [=] Result: 5.0000
30 [=] Result: 10.0000

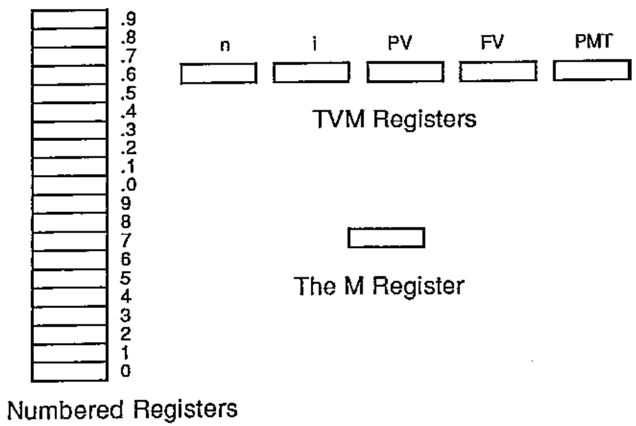
After reading to this point in your manual, you have learned the basics of using your EL-733A Financial Calculator. The next chapter explores FIN mode and financial problem solving.

Chapter 2. FIN Mode

When you place the EL-733A in FIN Mode by pressing [2ndF] [MODE] repeatedly until the FIN indicator comes on in the display, the active keys become those shown in this picture. All the financial and arithmetic functions are active, but the business and statistics functions are inactive.



How To Picture The Insides In FIN Mode



(The numbered registers must be activated before they can be used for storing and recalling numbers)

The EL-733A Memory In FIN Mode

In FIN mode, you gain access to 25 storage registers that have special uses in financial calculations. You can use these registers just to store numbers, but similarly to the business calculation registers (CST, SEL, MAR, and MU, described starting on page 25) in NORMAL mode, these 25 registers have special meanings to the calculator that allow you to perform powerful financial calculations and investment analyses.

The previous picture shows the names of all 26 registers that are available in FIN mode (including the M register). **When you first enter FIN mode, you have to activate the numbered registers (0 to 9 and .0 to .9) by pressing the [CFi]**

key once for every register that you wish to activate (up to 20), if you wish to use these registers for storage and recall.

Storing Numbers In FIN Mode Registers

The primary purpose of both the numbered registers and the TVM registers is to aid you in financial calculations. However, before we get into the discussion of financial problem solving, let's take a look at how you can use these registers for storing and recalling numbers.

To store a number in one of the TVM registers, for example the FV register, key in the number and press [FV]. To recall a number in one of the TVM registers, press [2ndF] [RCL] and then press the key that names the register ([n], [i], [PV], [FV], or [PMT]).

You can store and recall using the numbered registers, but first as described above, you need to activate them (press [CFi] at least 6 times before trying the following examples).

(Mode: FIN)

Example: Store 94.886 in register 5.

Solution: After activating the numbered registers up to register 5, press 94.886 [2ndF] [STO] [5]. If you complete these keystrokes and your display shows 94.8865, you are not in FIN mode!

Example: Store 11.5 ÷ 12 in the i register.

Solution: 11.5 [÷] 12 [=] [2ndF] [STO] [i] (there's more than one way to do this)

Example: Recall the number from register 5.

Solution: [2ndF] [RCL] [5]

The number 94.89 (in [2ndF] [TAB] [2] format) should be showing in your display. As long as you stay in FIN mode, the numbers in all the active registers will be preserved, even when you turn off the calculator. Those numbers will only change when you change them by storing or calculating a new number in that register or if you clear those registers by pressing [2ndF] [CA].

But storing and recalling is not the main subject of interest here. Now that you have the basic picture of FIN mode down, let's look at a fine discussion on the basics of...

Financial Calculations

A financial calculation is, in general, a calculation that involves money and time. Money accumulates interest as time passes, and, given some known situation, it is your job (along with our EL-733A) to determine the unknowns that may arise.

The EL-733A is equipped to calculate any unknown value that arises, provided it is correctly supplied with the knowns. Whether you want to calculate a mortgage payment or analyze an investment with irregular cash-flows, the EL-733A has all the tools you need to do the calculation.

Ensuring that you are using the tools simply and correctly is the goal of the explanation in the remainder of this chapter.

First, take a little time to look over the background information that follows. There are as many different financial languages as there are financial fields. So, this background information simplifies the financial language and some of the financial concepts down to pictures and the simple terms (abbreviations) that are printed on the face of the EL-733A.

SIMPLE INTEREST

There are two types of interest, simple and compound. Compound interest is the most common of the two in these modern times, and it is the type of interest that the EL-733A financial functions are designed around. Simple interest is rare, and it is not complicated mathematically, so there is no need for special functions on the calculator to handle it.

Because simple interest is rarely used anymore (except perhaps on personal loans from relatives and other special situations), it is not a big subject in this book. However, simple interest is closely related to compound interest, and it is definitely worthwhile to spend a few moments for a brief examination of simple interest.

If you were to borrow $1'000 for three years at 15% simple interest, at the end of those three years you would be responsible for paying back $1'150 (150 is 15% of 1000). Generally, simple interest is not very time dependent. The term and the amount of interest are negotiated and set at the beginning of the contract.

But simple interest is not very flexible. What if one year down the road you wanted to pay back $250 of that $1000 loan? Shouldn't that reduce the amount of interest you are responsible for paying? Well, if you start making accommodations for things like early payments, you are introducing a time dependence. The amount of interest you owe depends on the amount of time you hold the money. At that point, you are getting close to defining compound interest.

HOW COMPOUND INTEREST WORKS

Compound interest is interest that accumulates at a predefined rate on a periodic basis. If you deposit some money in a passbook savings account at a bank, chances are that each month a certain amount of interest will be added to that account, INCREASING the account balance. The amount of interest you receive each month depends on the balance of the account during that month **including interest that has been added in previous months.** Interest earns interest. And that is why it's called "compound interest."

Compound interest is **periodic.** It is very committed to a certain period of time. To borrow from a well worn phrase: time, in financial calculations, is of the essence.

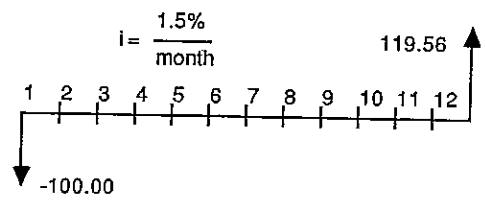
In fact, you must know the compounding period of a loan or investment before you start. This is one variable about an investment that has to be a known, because the whole calculation is based on it. The compounding period is usually specified or assumed (very often it's monthly).

Once you know the compounding period of a loan or investment, you also have to know that the compounding period has one interest rate associated with it. You don't have to know what the rate is (it can be your unknown), but have to recognize that it's the interest rate associated with the compounding period that causes money to change in value at the end of each period.

But enough generalizations. Let's look at something a little more pictorial.

THE CASH-FLOW SCHEDULE

This is a cash-flow schedule. It represents an investment in a savings account.

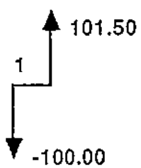


One hundred dollars is invested at 1.5% per month (18% APR compounded monthly) and left for one year. At the end of that year, when it is withdrawn, it has grown to 119.56. (Notice that the 18% APR here is just the monthly rate multiplied by 12. This is sometimes called the "nominal APR.")

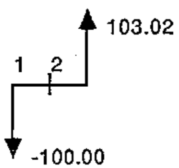
Understanding the cash-flow schedule is important when doing any type of financial calculation. The more complicated the calculation, the more necessary it is to draw a cash-flow schedule so that you have a clear picture in front of you when you key in the numbers.

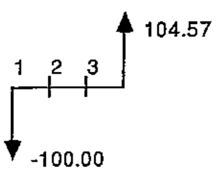
Notice on the previous cash-flow schedule that the periods are all regular (monthly) and that the interest rate per month is specified. Now to get a picture of how interest compounds, here are three cash-flow schedules that are shortened versions of the $100.00 savings investment.

These schedules show you what that $100.00 has grown to after one month, two months, and three months.



$$i = \frac{1.5\%}{\text{month}}$$





The calculation of the interest earned in the first month is easy. It's just a simple 1.5% of 100.00 or 1.50. So the new balance at the end of the first month is 101.50. Now, at the end of the second month, the calculation is just as easy, but the balance during the month is 101.50. So, 1.5% of 101.50 is 1.52. Add this to the account, and the new balance during the third month becomes 103.02.

To calculate the interest accumulated during the third month, start with the account balance of 103.02. 1.5% of 103.02 is (according to your EL-733A) 1.55. So the balance at the end of the third month is $104.57.

As you can see, compound interest means that interest is earned on interest earned previously. At the end of each period, interest is calculated based on the balance during that period, including the interest earned along the way.

If you break down a compound interest contract into individual periods, it looks like a string of simple interest loans. The amount owed is calculated at the beginning of the period and an amount of interest is agreed upon for the right to keep that money until the end of the period. Then at the end of the period, the interest is added in and a new simple interest loan is negotiated for the next period.

A **periodic** payment (addition to or subtraction from the balance) does not complicate this compounding process. As long as you know the balance at the beginning of the period and the periodic interest rate, calculating the amount of interest to add at the end of the period is a simple percentage calculation.

RULES FOR DRAWING CASH-FLOW SCHEDULES

Most of the cash-flow schedules that you draw will be quick sketches that help you visualize an entire problem before you key it in. This manual uses cash-flow schedules throughout its explanations and examples in financial calculations. A few simple rules are listed below that you should keep in mind when you are sketching the cash-flow schedule to start solving a financial problem.

1. The horizontal direction on a cash-flow schedule represents time. Time is divided up into regular periods (daily, monthly, quarterly, or annual periods are common). Before you key the numbers into the calculator, the compounding period for the interest has to be the same as the period shown on your cash-flow schedule.

2. The vertical up- and down-arrows represent cash-flows. Up-arrows represent positive cash-flows, which is money coming in. Down-arrows represent negative cash-flows, which is money going out.

3. Cash-flows can occur only at the beginning or ending of each period. Two or more cash-flows occurring at the same time must be netted together before they can be keyed into the calculator. This means that, with the exception of the first and last periods, only one cash-flow can occur per period.

As long as your cash-flow schedules conform to the above three rules, you will be able to key them directly into your EL-733A for a solution. Often times, especially in complicated cash-flow situations, a little work may be involved in making a cash-flow schedule conform to those three rules. So keep reading.

THE THREE PARTS OF A FINANCIAL SOLUTION

The solution to every financial problem consists of three general steps: (1) deciphering the jargon into terms that you understand, (2) drawing the cash-flow schedule (or picturing it in your mind), and (3) crunching the numbers.

Now the EL-733A takes a very active role in the crunching the numbers part. It will give you accurate answers based on the information that you supply. Also, you may make some use of the EL-733A when constructing the cash-flow schedule. With situations like wrap-around mortgages and discounted contracts, it may take several Time-Value-Of-Money calculations to even arrive at the final cash-flow schedule.

However, the EL-733A is admittedly weak when it comes to deciphering the jargon of a particular financial field into terms like n, i, PV, FV, PMT, or CFi and Ni. The responsibility of translating the words into the right numbers in the right place falls primarily on your shoulders, and this is often the most difficult part of coming up with exactly the right answer. In the explanation and examples that follow, you will get a little practice at deciphering the jargon.

The EL-733A Financial Functions And Terms

The EL-733A has two main groups of financial functions: the Time-Value-Of-Money functions (TVM functions) and the Discounted Cash-Flow analysis functions.

The TVM functions are designed to work with problems that have a cash-flow at the beginning of the time line, a cash-flow at the end of the time line, and a stream of regular, level cash-flows in between. (The amount of any cash-flow can be zero.)

The Discounted Cash-Flow functions can usually be used to solve any problem that cannot be solved with the TVM functions.

<u>RECOGNIZING A TVM PROBLEM</u>

The primary TVM functions are listed below:

[n] Number of periods.

[i] Interest rate per period.

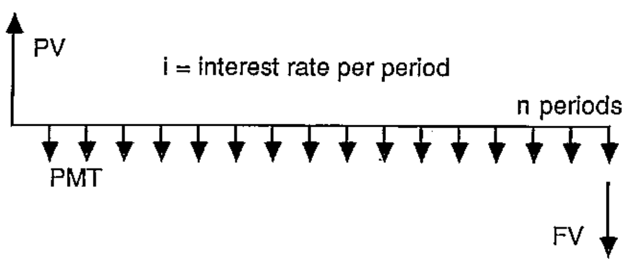
[PV] Present value.

[FV] Future value.

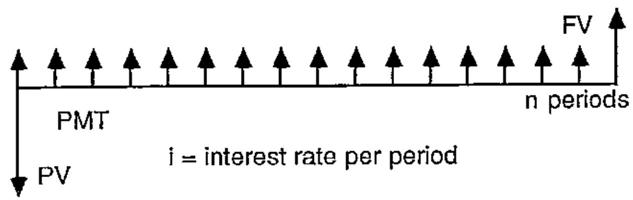
[PMT] Payment.

Other functions are provided for amortization and interest conversions, but those functions are addressed starting on pages 93 and 69 respectively.

Financial problems that work with the TVM group of functions usually have cash-flow schedules that look like this:



...or this:



They have one cash-flow at the beginning of the time line (called the PV or Present Value), one at the end of the time line (the Future Value or FV), and a stream of regular periodic payments (PMT), all of the same amount, in between. Typically, mortgages and loans, leases, savings, annuities, and contracts with regular payments can be analyzed using the TVM group of functions.

RECOGNIZING A DISCOUNTED CASH-FLOW PROBLEM

Four functions on the EL-733A deal with Discounted Cash-Flow Analysis. These functions are listed below:

[CFi] Cash-flow group i (where i can be 0, 1, 2, etc.).

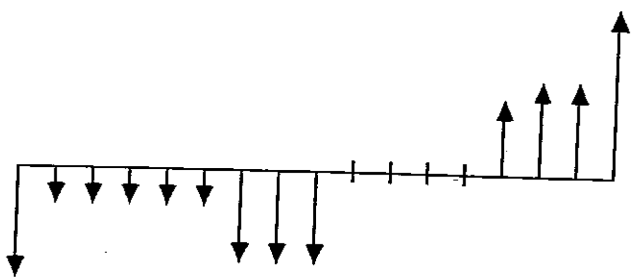
[Ni] Number of cash-flows in group i.

[NPV] Net present value.

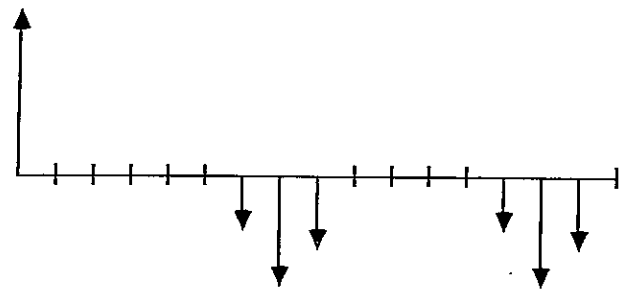
[IRR] Internal rate of return.

Financial problems that work with the Discounted Cash-Flow Analysis functions can have cash-flow schedules that look like just about anything, as long as they conform to the three rules for cash-flow schedules on page 39.

A problem that requires the Discounted Cash-Flow Analysis functions may have a cash-flow schedule that looks like this:



...or this:



In fact by using the Discounted Cash-Flow Analysis functions, just about any investment situation can be described to your calculator and analyzed. As long as the periods are regular and you understand that interest compounds once per period, you will find these functions to be extremely flexible.

Up to this point, the description of financial problem solving has been heavy on the theory and light on the examples. If you have been following along since the beginning of this chapter, you have probably pressed just one or two keys on the EL-733A since you started.

But now that you have read through the necessary background information, the next section starts in with some examples of TVM calculations. TVM problems are usually the most familiar and the easiest to understand. And understanding TVM problems is a good prerequisite for Discounted Cash-Flow Analysis problems, which are dealt with in the section that starts on page 105.

TVM Applications

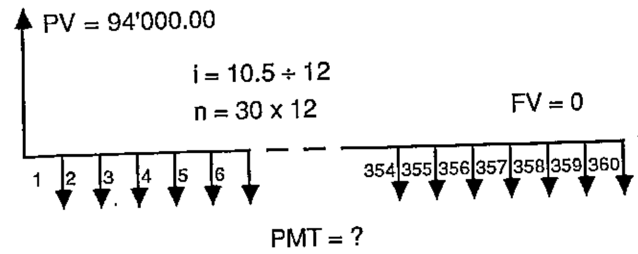
In working a TVM problem, you have to translate the financial language that you are used to dealing with to the simple language of the five TVM keys. When it comes to TVM problems, the calculator understands only the terms n, i, PV, FV, and PMT. All of the language that you may be used to working with (balloon payment, residual, points, coupon, the list goes on and on) has to be translated into these five terms.

If you truly understand your particular financial language and if you know how to draw a cash-flow schedule based on that language, the translation is easy. Let's give it a try.

A TYPICAL MORTGAGE (COMPUTING A PAYMENT)

Example: As a realtor, you have a chance to sell a $106'000.00 house to a buyer that you have been showing houses to for the last couple of weeks. The buyer can come up with about $12'000.00 as a down payment, leaving about $94'000.00 to finance. The interest rate is hovering at around 10.5% APR. The term of a typical mortgage is 30 years. What will be the payment on this loan?

Explanation: The cash-flow schedule of this problem is an easy one to draw. It is drawn here from the perspective of the borrower. To the borrower, the payment will be money going out each month which makes it a down-arrow (a negative value) on the cash-flow schedule. It is important, when drawing a cash-flow schedule, to pick one perspective, either that of the borrower or that of the lender, and to stick to that perspective throughout the problem.



A couple questions may arise when you look at the above cash-flow schedule. First, how do you know that the period is monthly; where was that stated? And second, what is an APR, and how do you know that it needs to be divided by 12 in this case.

THE [2ndF] [X12] KEY, MONTHLY PERIODS

The answer to the first question is that the period should be stated in the description of the problem. However, usually if the period isn't explicitly stated in a description, you can assume it is monthly. In fact, the monthly period is so common that the second function [2ndF] [X12] is provided above the [n] key on the EL-733A to speed up the conversion of years into months.

THE [2ndF] [÷12] KEY AND APR

The answer to the second question is that lending institutions (most banks, the FHA, and finance companies) usually quote interest as a "nominal APR (Annual Percentage Rate)." They take the periodic rate that they use in their calculations and they multiply it by the number of periods in a year (usually 12). So usually the first thing you need to do given an APR with monthly compounding is to divide it by 12. Notice that the [2ndF] [÷12] function is provided above the [i] key for this exact purpose. *The periodic rate is the only one that makes any sense to the calculator.*

Also, along with the nominal APR that they divide by 12 and use in their calculations, most lenders are required to quote the effective APR or true APR which is a calculated annual rate that includes compounding (and finance charges). For now, don't try to use effective rates in your calculations. Calculating effective rates is covered on pages 72 and 90.

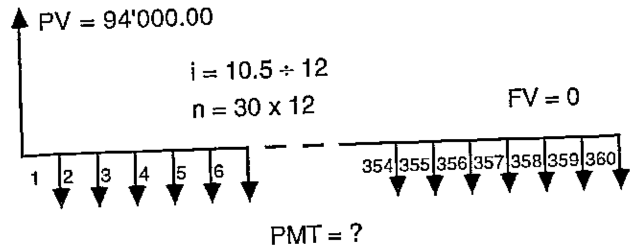
PAYMENTS AT THE BEGINNING OF THE PERIOD

Your EL-733A can be set to solve TVM problems with payments at either the beginning or the end of the cash-flow period. The only reason that this is mentioned here is because if you have your calculator set to BGN mode, you **will not be able to get the right answer** in this example. The [BGN] key is used to switch the calculator in and out of BGN mode. When the display indicator BGN is turned on, the calculator is set to solve TVM problems assuming payments occur at the beginning of the period.

Why should it make any difference if the payments occur at the beginning or at the end of the period? Well, think about it.... The quicker the balance is reduced, the less interest is going to accumulate and the smaller the payment.

So in this problem, be aware that **your calculator should not be in BGN mode when you are solving for this mortgage payment.** BGN mode is discussed more on page 56.

Take another look at the cash-flow schedule for this example:



The amount financed is $94'000 (PV = 94'000) to be completely paid off (FV = 0) over a period of 30 years or 360 months at a periodic interest rate of 0.875% per month. The information is all there and, after reading the explanation to this point, you should understand most of the subtleties about this mortgage problem.

The following keystrokes solve for the payment in the above cash-flow schedule. Make sure the BGN indicator is **not** on in your display:

Keystrokes: 106'000 [−] 12'000 [=] [PV]
10.5 [2ndF] [÷12] [i]
30 [2ndF] [X12] [n]
0 [FV]
[COMP] [PMT] Result: −859.85

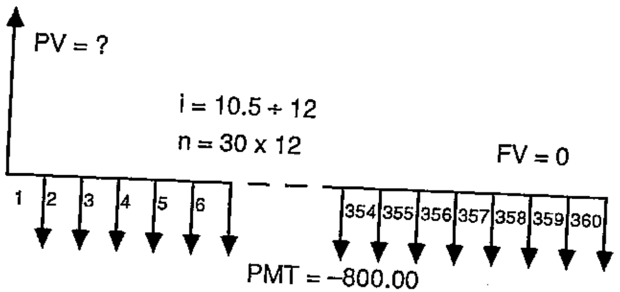
Notice that the [2ndF] [÷12] and [2ndF] [X12] functions are separate functions above the [i] and [n] keys provided for your convenience. They replace the keystrokes [÷] 12 [=] and [X] 12 [=], respectively.

PV CALCULATIONS

Example: Referring to the last example (and with your calculator set to FIN mode and the display at [TAB] [2]), the buyer tells you that an affordable payment on the mortgage loan (which doesn't include taxes and insurance) would be right around $800.00. What does the price of the house have to be to reduce the payment from $859.85 to 800.00?

Keystrokes: 800 [+/-] [PMT] [COMP] [PV]
[+] 12'000 [=]   Result: 99'456.61

Explanation: The new situation looks like this on a cash-flow schedule:



Now you know the payment (PMT is specified to be –800.00), and you need to calculate a new present value (PV). To this new PV, you need to add the amount of the down payment to arrive at the desired price of the house.

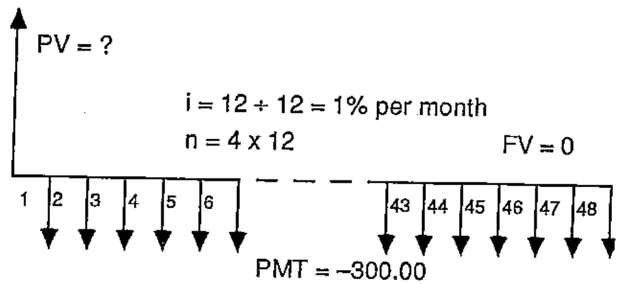
Once you have a TVM situation keyed in, if you want to make just one change to see how it affects another value,

you don't have to rekey everything. That is the real beauty of playing "what if" on the EL-733A. You can ask yourself questions like, "What if the payment changes to 800.00; how does that affect the PV?" and, "What if the interest rate changes to 11.2% APR; how does that affect the payment?" And the answers to those questions are just a few keystrokes away.

EXAMPLES OF PV AND PMT CALCULATIONS

Example: You are in the market for a new car. You are going to trade in your old car and you can afford about a $300 per month car payment . The interest rates are at about 12% APR. What can you afford to borrow on a car if you get a 4 year loan? How about a 5 year loan?

Solution: The cash-flow schedule for the 4 year loan looks like this:



The amount (PV) that you borrow will be completely paid off (FV = 0) in 48 months (n = 48) at $300.00 per month (PMT = –300.00). The interest rates you estimate to be around 12% APR or 1% per month. With your calculator in FIN mode and your display set to [2ndF] [TAB] [2], here are the keystrokes. Make sure BGN is **off**:

300 [+/-] [PMT]
0 [FV]
1 [i]
4 [2ndF] [X12] [n]
[COMP] [PV] Result: 11'392.19

Now how much more can you afford to borrow if you go for 5 year financing? Since you have the problem all keyed in, just change the one value (n) and recalculate PV:

5 [2ndF] [X12] [n]
[COMP] [PV] Result: 13'486.51

Example: You finally find a car that you like with a price tag of $23'000.00. You can expect to get about $3'600 trade in on your old car. What will your monthly payments be on a 5 year loan?

Solution: Again, there's only one value (PV) that you need to change, and then you need to recalculate the payment.

23'000 [−] 3600 [=] [PV]
[COMP] [PMT] Result: −431.54

Example: You luck out and find some financing at 10.5% APR. How does that affect your payment?

Solution: Just key in the new interest rate (make it a periodic rate) and then solve for the new payment. Here are the keystrokes:

10.5 [2ndF] [÷12] [i]
[COMP] [PMT] Result: −416.98

Can your budget handle this slight extension for the car you want? No problem, right?

You can see that the EL-733A is a handy tool to take with you whenever you are on the way to buy a car or to make some purchase that will involve financing.

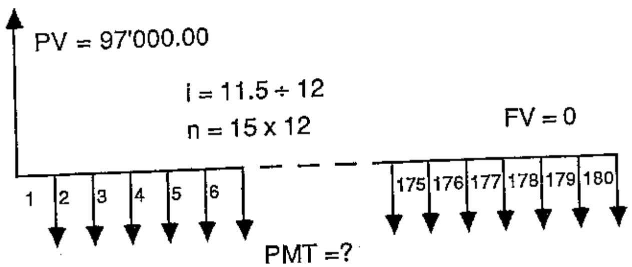
But we haven't yet covered all the aspects of TVM problems. We have only covered PMT and PV calculations to this point, and this is just a scratch on the surface of financial calculations.

BALLOON PAYMENTS AND [FV] CALCULATIONS

On the EL-733A, the future value ([FV]) is an amount left at the end of the cash-flow schedule that is separate from any regular payment that may occur at that same point. In the case of a balloon payment, which is a payment at the end of a loan contract that completely pays of the remaining balance, it is important to recognize that the result for FV may have to be added to a payment amount to determine the actual final payment amount on the loan.

Example: As a construction engineer, you recently got a five year contract job in Utah overseeing the construction of a modern, clean, coal-fired power plant. You were able to purchase a nice house with a 15 year mortgage of $97'000.00 at an interest rate of 11.5% APR. However because your job is on a five year contract, a balloon payment is scheduled at the end of those five years to pay off the balance of the loan. What is the mortgage portion of your house payment (not including taxes and insurance) and what is the amount of the balloon?

Solution: The first part of this solution is a payment calculation. Your sketch of the cash-flow schedule of the 15 year mortgage should look something like this:



And the keystrokes are:

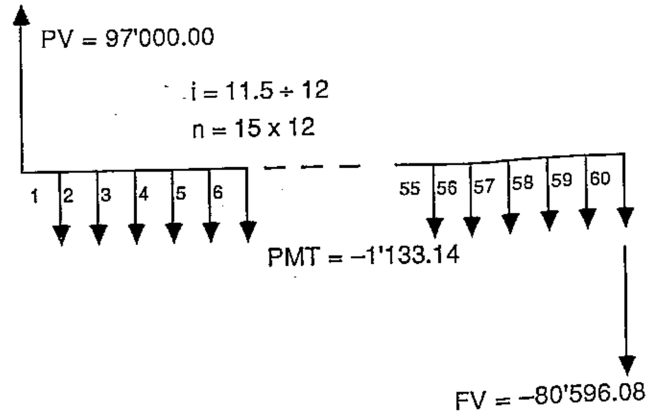
(Mode: FIN)

15 [2ndF] [X12] [n]
11.5 [2ndF] [÷12] [i]
97'000 [PV] 0 [FV]
[COMP] [PMT] Result: –1'133.14

Once you have calculated the payment, the FV calculation is just a matter of changing [n] to 60 (5 years x 12 months) and calculating [FV].

5 [2ndF] [X12] [n] [COMP] [FV] Result: –80'596.08

That is the amount left to pay on the loan after the 60th payment. But look at the cash-flow schedule of the five-year scenario with the balloon payment:



The actual final balloon payment is going to include the last regular monthly payment. So the final balloon payment is:

[2ndF] [RCL] [PMT] [+]
[2ndF] [RCL] [FV] [=] Result: –81'729.22

Recalling numbers from the TVM registers is described on page 33. Also, do you remember why the payment results are negative? Money paid out is always negative. That is the convention used by the calculator.

PAYMENTS IN BGN MODE

BGN mode was mentioned briefly on page 48. To put the EL-733A in BGN mode, press the [BGN] key. An indicator should come on in the display to tell you the calculator is set to BGN mode.

BGN is short for BEGIN. When set to BGN mode, the calculator assumes that payments ([PMT]) in TVM problems occur at the beginning of the period rather than at the end. BGN mode only affects the TVM functions and does not have any affect on the Discounted Cash-Flow Analysis functions ([CFi] , [2ndF] [Ni] , [NPV] , and [IRR]).

Payments at the beginning of the period are sometimes called "payments in advance" or "annuity in advance." Payments at the end of the period are sometimes called "payments in arrears" or "annuity in arrears."

LEASES

In leases of property or equipment, it is common to have the payments at the beginning of the period. A lease consists of a loan of something of material value in exchange for periodic payments. You can lease a car, or you can lease equipment for your business, or you can lease a house or any real property.

Lease contracts can vary considerably depending on what is being leased and on the intentions of the parties involved in the lease. Many leases of cars or equipment are written as purchase agreements: a fancy form of financing. Leases of property and buildings tend to be written more as rent/time agreements; one party agrees to make monthly payments on a property for a certain amount of time and the other party agrees to let them stay at that property for that amount of time.

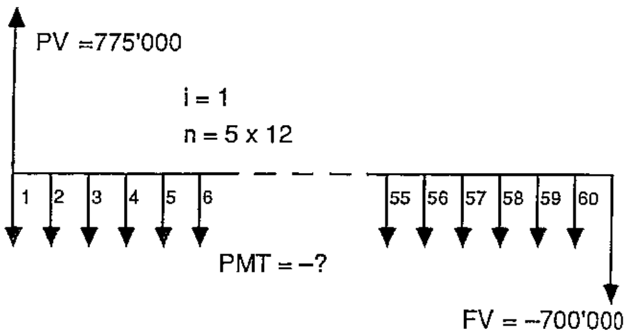
Leases that require payments up front (for example, the last three payments up front or the first year's payments up front), or that have some other alterations to the steady, even payment stream may require some finagling before they can be solved using the TVM functions. Leases that get overly complex often must be tamed by using the Discounted Cash-Flow Analysis functions.

An example of a straightforward lease appears below and an example of a more complicated lease appears on page 85. To solve this example, your EL-733A must be set to BGN mode (the BGN indicator is on in the display). To set the display to show dollars and cents, press [2ndF] [TAB] [2].

Example: As the president of a start-up company specializing in solar-powered water pumps that take advantage of the properties of the newly developed "thermo-memory metals," you have brought considerable success to the firm. A move to larger facilities is the next necessary step, and you are negotiating a five-year lease on a prospective building.

The building you would like to lease is valued at $775'000.00, and it's in an area that will allow for good growth, so you are negotiating an option to buy at the end of the lease. After weeks of negotiation, you and the owner settle on a buyout price of $700'000.00. The owner wants to see about a 12% yield on the value of the building during the five-year lease. What are your monthly payments?

Solution: Here's the situation illustrated on a cash-flow schedule:



The cash-flow schedule above is drawn according to your perspective. You move into the building on day-one, so the value of the building shows as a positive. You are responsible for making the the payments, so they show as a negative (including the buyout figure at the end).

As usual, once you boil the words down to a cash-flow schedule, the keystroke procedure is not very involved. Notice that the first payment on the cash-flow schedule occurs at the beginning of the time line. That means this calculation requires BGN mode:

(Mode: FIN)

[BGN] (to turn on the BGN display indicator)
775'000 [PV]
5 [2ndF] [X12] [n]
1 [i]

700'000 [+/-] [FV]
[COMP] [PMT] Result: –8'582.51

Notice that the order in which you input the values of n, i, PV, PMT, and FV is not important. All you are doing when you press one of those keys is storing a number in a register. Only when you precede one of those keys by pressing [COMP] does the calculator do any computing. When you compute one of the five TVM values, the calculator bases its answer on the numbers in the other four TVM registers.

[i]: INTEREST RATE CALCULATIONS

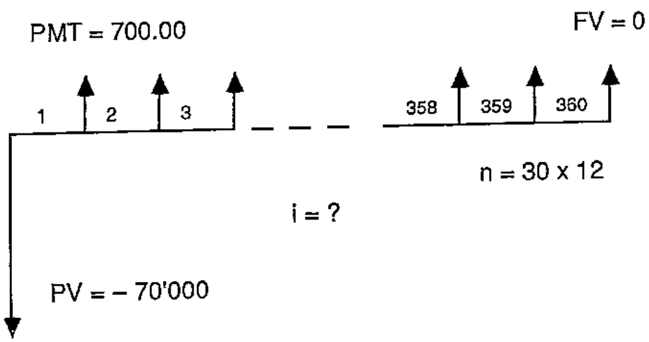
The interest rate on a loan or investment with a regular payment stream can be calculated by pressing [COMP] [i] . If the cash-flows are irregular or uneven, the Discounted Cash-Flow Analysis function [IRR] (described starting on page 115) is generally required for a solution.

Interest calculations are typically used to answer questions that begin "what return am I getting if...?" or "what does the interest rate have to change to in order for...?" or "what is the real interest rate on this...?" It is a valuable function that often is not taken to its full advantage.

The [i] key always expects and computes a periodic rate. If you want to see the APR, you have to multiply the result by the number of periods in a year (12 for monthly periods, 4 for quarterly periods, etc).

Example: You are selling one of your rental properties for a price of $75'000. A propective buyer approaches you with $5'000 for a down payment and the ability to pay taxes, insurance, plus $700 dollars a month. Is it likely that this person will be able to find the financing to cash you out?

Solution: If an institution were to loan your prospective buyer the cash, this is the situation that institution would be looking at (from the lenders perspective).



Would the return (or interest rate) be high enough in this case? If the calculated rate is about what the market is currently bearing, you may have a buyer for your house.

First, make sure your calculator is **out of** BGN mode by pressing [BGN] to make the BGN indicator turn off. Then press:

(Mode: FIN)

30 [2ndF] [X12] [n] 0 [FV]
70'000 [+/-] [PV]
700 [PMT]
[COMP] [i] Result: 0.97

This is the monthly rate. To compare it with the advertised rates multiply by 12 to get a nominal APR of 11.63%. At the time of this writing, that is a reasonable rate on a mortgage (perhaps even a little high) so your prospective buyer will likely be able to get financing.

"POINTS UP FRONT" (PREPAID FINANCE CHARGES)

Nowadays, it is almost the norm to have to pay some finance charges up front in order to secure a loan, especially a mortgage. In the U.S., the FHA (Federal Housing Act) rates are well known for their dependence on the "points" that you are willing to pay up front. These "points" are percentage points. The more percentage points of the borrowed money you are willing to pay at the onset of the loan, the lower the rate will be that is used to calculate your payment.

Prepaid finance charges or "points up front" have the effect of increasing the actual interest rate that is paid on the borrowed money. These prepaid finance charges reduce the net amount of money borrowed up front, without reducing the payment. This is not the same as a "down payment," which reduces both the money borrowed and the payment amount.

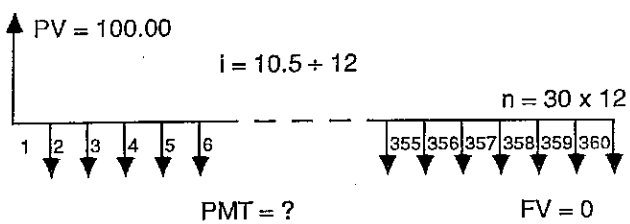
Example: As a real estate sales person, you try to keep up on the FHA rates from week to week. Today you were told that the FHA terms were as follows:

10.5% with 1/4 point up front,
10.0% with 1/2% up front,
9.5% with 2.5% up front, and
9.0% with 4.25 points up front.

These rates apply to fixed rate, thirty-year loans. Payments and compounding are monthly. Payments are in arrears.

(1) By how much do those "points up front" increase each of the quoted rates? (2) What do the points up front amount to on a $90'000 loan, and what are the payments on a $90'000 loan in each case?

Solution: The first of the above two questions is answered most easily by looking at a $100.00 loan. The amount of the loan does not matter because the result that you are after is an interest rate. Ask yourself the question, what would be the payment on a 30 year, $100.00 mortgage at 10.5% APR. The cash-flow schedule follows:



This is just a payment calculation. Because the payment occurs at the end of the period, make sure that the BGN indicator is **not** on in the EL-733A display. Also, check to make sure the FIN indicator is on in the display telling you that you are in FIN mode:

(Mode: FIN)

100 [PV]
10.5 [2ndF] [÷12] [i]
30 [2ndF] [X12] [n]
0 [FV]
[COMP] [PMT] Result: –0.91

It would take payments of 91¢ a month to pay off a $100.00, 10.5% loan in 30 years. But that is not the question. The 1/4 point up front requirement actually reduces the net amount of money borrowed by 25¢, right? But the payments don't change, which means the actual APR is a little higher. Here are the keystrokes:

[2ndF] [RCL] [PV] [–] .25 [%] [PV]
[COMP] [i] Result: 0.88

The computation of [i] takes a few moments. Interest is not something that can be solved for directly, so the calculator has to use a numerical guessing game to arrive at an answer. The result here, 0.88 is a periodic rate. Multiply by 12 to get an APR:

[X] 12 [=] Result: 10.53

The 1/4 point up front boosts the APR by about 0.03%. What about the other three cases? The following sets of keystrokes give the three resulting yields:

For 10.0 with 1/2% up front:
10.0 [2ndF] [÷12] [i]
100 [PV]
[COMP] [PMT]
[2ndF] [RCL] [PV] [−] .5 [%] [PV]
[COMP] [i] (pause)
[2ndF] [X12] Result: 10.06

For 9.5 with 2.5% up front:
9.5 [2ndF] [÷12] [i]
100 [PV]
[COMP] [PMT]
[2ndF] [RCL] [PV] [−] 2.5 [%] [PV]
[COMP] [i] (pause)
[2ndF] [X12] Result: 9.79

For 9.0 with 4.25% up front:
9 [2ndF] [÷12] [i]
100 [PV]
[COMP] [PMT]
[2ndF] [RCL] [PV] [−] 4.25 [%] [PV]
[COMP] [i] (pause)
[2ndF] [X12] Result: 9.49

The next question you need to answer is what are the numbers (payment and points) on a $90'000 mortgage?

You have already had a little practice doing percentage and payment calculations, but, at the risk of making you a little restless, we will go through the explanation of the last case (9.0% APR with 4.5% up front) anyway.

The finance charge is 4.5% of $90'000:

90'000 [X] 4.5 [%] Result: 4'050.00

So a buyer who wishes that the payment be based on 9.0% APR will have to come up with a $4'050 prepaid finance charge above and beyond the down payment!

The payment calculation should be no problem. The keystrokes below assume that you have been following this solution from the beginning, so the TVM registers are all set up for a 30 year mortgage. You only need to change [i] and [PV]:

90'000 [PV] 9 [2ndF] [÷12] [i]
[COMP] [PMT] Result: −724.16

Now you should be able to work through the payment and points up front calculations for the remaining three cases (10.5% APR with 1/4 point up front, 10.0% APR with 1/2 point up front, and 9.5% APR with 2.5 points up front). The keystrokes and results follow for your reference.

For 10.5 with 1/4% up front:

[2ndF] [RCL] [PV] [X] .25 [%] Result: 225.00
10.5 [2ndF] [÷12] [i]
[COMP] [PMT] Result: −823.27

For 10.0 with 1/2% up front:

[2ndF] [RCL] [PV] [X] .5 [%] Result: 450.00
10 [2ndF] [÷12] [i]
[COMP] [PMT] Result: −789.81

For 9.5 with 2.5% up front:

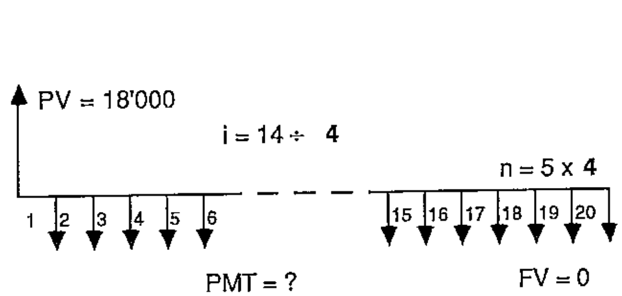
[2ndF] [RCL] [PV] [X] 2.5 [%] — Result: 2'250.00
9.5 [2ndF] [÷12] [i]
[COMP] [PMT] — Result: –756.77

This verifies what you already know about FHA financing, the more points up front you are willing to pay, the smaller the payment.

Example: A finance company is charging finance charges up front on loans of up to $25'000 **with quarterly payments** in arrears. The interest rate they use to calculate the payment is 14% APR **compounded quarterly** and the term is negotiable. The finance charge is 3/4% per year of the contract. You are interested in a five year loan of $18'000 for home improvement. What would your payments be if you chose to go with this finance company, and (including the prepaid finance charges) what periodic rate would they be earning on the money they loan to you?

Solution: As you saw in the previous example, whenever you are dealing with prepaid finance charges, if you wish to compute the actual interest rate, you have to do it in two steps. The first step is to compute the payment without considering the prepaid finance charges. The second step is to compute the actual interest rate considering the loaned money less the finance charges as the actual Present Value.

The cash-flow schedule for the first step, which is the payment calculation, is as follows:



The keystrokes for a payment calculation should be looking at least vaguely familiar by now. However, because this loan calls for quarterly payments, you will not use the [X12] and [÷12] functions:

(Mode: FIN)

18'000 [PV]
14 [÷] 4 [=] [i]
5 [X] 4 [=] [n]
0 [FV]
[COMP] [PMT] — Result: –1'266.50

So your quarterly payment would be $1'266.50, which is a fairly round number. Now, the prepaid finance charges you have to pay depend upon the length of the loan. You wish to borrow the money for five years, and with a finance charge of 3/4% per year, the calculation looks like this:

[2ndF] [RCL] [PV] [X] .75 [%]
[X] 5 [=] — Result: 675.00

Subtract this finance charge from the borrowed amount and calculate the actual periodic rate:

[X↔M] [2ndF] [RCL] [PV]
[−] [RM] [=] [PV]
[COMP] [i] Result: 3.93

To compute a nominal APR from this actual periodic rate, multiply by four (there are four quarters in a year):

[X] 4 [=] Result: 15.71

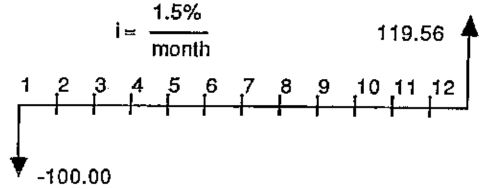
EFFECTIVE INTEREST RATES

Interest rate terminology can be confusing. There are "periodic rates," "nominal APR's," "actual APR's," "effective APR's," "actual effective APR's," "variable rates," "blended rates," "coupons," "yields," "returns," "finance charges," and a sea of other terms that depend upon who you are talking to and what field of finance you are discussing.

As you wade through this sea of terminology, ask as many questions of the people you are dealing with as you must to clarify the situation and to draw a cash-flow schedule. Keep in mind that the most important rate to know for financial calculations is the periodic rate. This is the rate that regulates how money grows from one period to the next. The other rates that are quoted are always calculated starting with the periodic rates.

One common way to quote an APR (annual percentage rate) is to multiply the periodic rate by the number of periods in a year. But as described earlier (page 47), this "nominal APR" is more of a convenient approximation of what actually happens than an accurate reflection of the interest paid. It does not incorporate compounding, which can be significant at the higher interest rates.

The effective APR is an annual percentage rate that does incorporate compounding. On that $100.00 savings account back on page 37, the nominal APR for the account was 18%, but the balance of the account after compounding this rate for a year was $119.56. By the fact $19.56 was earned on $100.00 in one year, you can say that the effective APR during that year was 19.56%. By compounding an 18% APR monthly, you boost the effective APR to 19.56%:



Two functions on the EL-733A allow you to convert APR's to EFFective APR's and vice versa:

- The [2ndF] [→EFF] key allows you to calculate an effective annual rate given a nominal APR and the number of periods in a year.

- The [2ndF] [→APR] key allows you to calculate a nominal APR given an effective annual rate and the number of periods in a year.

The keystrokes required to calculate the above effective rate of 19.56% for 12 compounding periods of an 18% APR would be as follows:

12 [2ndF] [→EFF] 18 [=] Result: 19.56

To convert this rate back to an APR, first store it by pressing [X→M], then press the following keys:

12 [2ndF] [→APR] [RM] [=] Result: 18.00

Remember: key in the number of compounding periods in a year first, press the conversion keys, then key in the rate you wish to convert and press [=].

The following examples are provided for practice in converting rates from APR's to effective APR's. This is an important skill to develop before looking at the subject in the next section *(Payment Period And Interest Period Differ).*

Example: What is the effective annual interest rate for 18% APR, compounded daily?

Solution: The first question you need to ask when you are dealing with daily compounding is, "how many days are in a year?" The answer may not be as simple as it seems. Some contracts are written with daily compounding based on a 360 day year and some are based on a 365 day year. And neither of these are accurate per se, because a calendar year varies from 365 to 366 days a year.

In this problem, assume that a year is considered to have 365 days. The keystrokes are as follows:

(Mode: FIN)

365 [2ndF] [→EFF] 18 [=] Result: 19.71642428

All ten digits of the result are shown because everyone is important. You probably have your calculator set to display just two decimal places, but whenever you are working with interest rates (especially), all the digits are important.

Press [2ndF] [TAB] [9] to display all the digits. If you write down an answer to be used in a later calculation, copy down every digit available. If you have your display set to [2ndF] [TAB] [2] and you use that displayed version of the result, you may be introducing rounding errors to your succeeding calculations. It is best to try and keep intermediate results in the calculator's memory until you reach your final answer, which you may round as you please.

Example: What APR, compounded quarterly, yields an effective annual percentage rate of 21.5%.

Solution: 4 [2ndF] [→APR] 21.5 [=] Result: 19.96

PAYMENT PERIOD AND INTEREST PERIOD DIFFER

How do you deal with daily compounding and monthly withdrawals? Or monthly compounding and quarterly payments? One of the fundamental values in any financial calculation on your EL-733A is the period. There can only be one period, so the interest compounding period has to match the payment period.

But in the real world they often don't match. Where do you start?

Well, the only thing you need to understand to handle problems where the payment and interest periods don't match is that **two different interest rates compounded on two different periodic schedules are not considered different in a financial calculation as long as they yield the same EFFECTIVE rate over the same period of time.**

When the periodic interest rate is compounded on a period other than the payment period, it must be changed to a different periodic interest rate that compounds on the payment period but that yields the same effective rate. Here's an example to show how that is done.

Example: You recently financed the purchase of an electronic optical french fry sorter for your food processing company by taking out a $200'000 loan with quarterly payments. The interest rate on the loan is 9.75% APR compounded monthly and the term of the loan is 5 years. What are your payments?

Solution: Start this problem as if it were just a standard payment calculation on a 5-year loan with quarterly payments:

(Mode: FIN)

5 [×] 4 [=] [n]
200'000 [PV] 0 [FV]

But before you key in the interest rate, you must first convert it to a quarterly rate that yields the same effective annual rate as 9.75% compounded monthly. To convert it to an effective rate, press:

12 [2ndF] [→EFF] 9.75 [=] Result: 10.20

Store that effective rate in the M register:

[X→M]

Then ask the EL-733A what APR compounded quarterly will yield that same effective rate:

4 [2ndF] [→APR] [RM] [=] Result: 9.83

That is the correct **APR** based on quarterly compounding. So divide it by 4 and store it as the **periodic** interest rate:

[÷] 4 [=] [i]

Then compute your payment on the loan. Press:

[COMP] [PMT] Result: –12'777.81

Along these same lines, an example of quoting an effective rate that includes all finance charges is shown on page 79 in the section *More TVM Examples.*

VARIABLE RATE LOANS

Variable rate loans started becoming more and more common during the skyrocketing interest rates of the early 1980's. If the economy is unstable or interest rates are wildly going up and down, it usually makes sense for both parties to allow the interest rate on a loan to vary. This practice can be beneficial to both parties in the loan because lenders feel more comfortable lowering their rates when times are good, if they know they have a safety valve written into the contract to use in case the market rates increase.

Variable rates complicate a TVM problem, because the TVM functions depend on an even payment calculated at a single interest rate. However, all is not lost. By breaking variable rate problems into separate problems, each corresponding to an adjustment in the interest rate, the solution becomes a series of simple TVM problems.

Some of the fairest variable rate loans are those written so that the interest rate is tied to some economic indicator (for example, the prime lending rate, or the rates on government bonds or treasury bills) that can not be controlled by either the lender or the borrower.

In the U. S., the most interesting value in a variable rate loan calculation is the payment, because the payment is the value that has to compensate for the fluctuating interest

rate. Most variable rate loans are written so that the rates can go up or down, which means the payment can go up or down. And there are usually limits on how much the interest rate can increase in one year and on the maximum interest rate.

Often with variable rate loans, an infinite number of possiblities exist for interest rate and, thus, payment amount. At the onset of the loan, a borrower or lender could spend many long nights in front of a calculator speculating on variations of the loan payment, but the only variation that is worthwhile looking at ahead of time is the (dreaded for the borrower) "worst case scenario," which is dictated by the limits of the contract.

The borrower has to be able to handle the payments that result from the "worst case scenario." That is, if everything falls apart and the interest rates in the loan head toward the ceiling as rapidly as the contract allows, will the payments still be affordable? If not, somebody is taking a gamble.

Solving for the payment schedule on a variable rate loan goes something like this:

1. Solve for the payment (amortize the loan) using the first interest rate in the contract.
2. Calculate the remaining loan balance (FV) at the first time the interest can increase, assuming the maximum increase.
3. Re-amortize the loan for the remaining term, using the balance calculated in step 2 as the present value (PV) and using the increased interest rate.

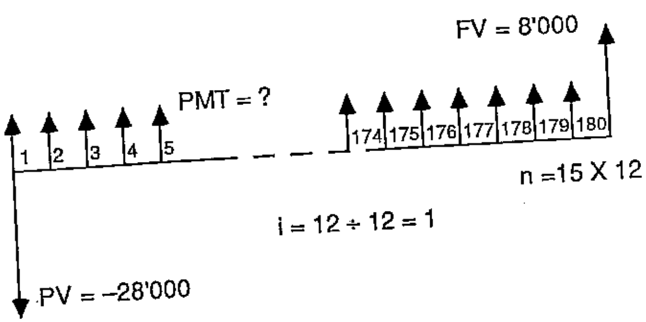
4. Calculate the remaining balance on the loan (FV) at the next time the interest can increase, again assuming the maximum increase.

5. Re-amortize the loan for the remaining term, using the balance calculated in step 4 as the present value (PV) and using the increased interest rate.

6. Repeat steps 4 and 5 until you reach the ceiling on the interest rate or until the end of the loan.

Example: A $28'000.00, 15 year loan is written on a variable rate contract with the rates tied to the T-bill. The current interest rate on the contract is 12% APR and the interest rate ceiling is 16% APR. The interest rate is re-evaluated every 12 months of the contract and it can increase by a maximum of 1% APR at each re-evaluation. Payments are monthly and **in advance**, and compounding is monthly. At the end of the 15 year term, an $8'000 balloon payment is due. What is the worse case payment schedule?

Solution: First look at the cash-flow schedule for the loan assuming the first interest rate stays constant:



This cash-flow schedule is drawn from the perspective of the lender. To the lender, the money loaned out is a negative cash-flow and the payments are positive cash-flows.

The problem states that the payments are in advance, which means the calculator must be set to BGN mode. **Press** [BGN] **to turn the BGN indicator on in the display.** Then, the keystrokes to calculate payment for the first 12 months are:

(Mode: FIN)

28'000 [+/-] [PV]
8'000 [FV]
1 [i]
15 [2ndF] [X12] [n]
[COMP] [PMT] — Result: 316.86

That was easy. However, this $316.86 payment applies to the first 12 months only. In the worst case, the interest rate will jump 1% at the end of those 12 months. Calculating the payment when the interest changes is like writing a new loan for a **smaller balance** and a **shorter term** (14 years).

To calculate the **smaller balance** (after 1 year) press:

12 [n] [COMP] [FV] — Result: 27'492.27

This is amortized out for **a term of only 14 years** at the new interest rate of 13%. Don't forget to rekey the $8'000 balloon payment at the end:

[+/-] [PV] 8'000 [FV]

14 [2ndF] [X12] [n]
13 [2ndF] [÷12] [i]
[COMP] [PMT] Result: 335.51

So the worst case payment for the second year of the contract is $335.51, up about $19.00 from the first year's payment.

This process needs to be repeated until you reach the maximum interest rate of 16%. The keystrokes and resulting payments are shown below:

12 [n] [COMP] [FV]
[+/-] [PV] 8'000 [FV]
13 [2ndF] [X12] [n]
14 [2ndF] [÷12] [i]
[COMP] [PMT] Result: 353.80

12 [n] [COMP] [FV]
[+/-] [PV] 8'000 [FV]
12 [2ndF] [X12] [n]
15 [2ndF] [÷12] [i]
[COMP] [PMT] Result: 371.69

12 [n] [COMP] [FV]
[+/-] [PV] 8'000 [FV]
11 [2ndF] [X12] [n]
16 [2ndF] [÷12] [i]
[COMP] [PMT] Result: 389.11

This last calculation was the payment for the fifth year of the contract. The interest rate has hit the 16% ceiling, so the payment, in the worse case, would never exceed $389.11.

MORE TVM EXAMPLES

This section contains a broad assortment of Time Value of Money examples. The index on page 166 will guide you to a specific catagory of example. Amortization (including [AMRT] , [P/B], and [ACC]) is covered in the following section, starting on page 93.

After reading and working the examples up to this point in the manual, you should have a broad understanding of TVM solutions. The examples in this section contain less explanation. The solutions consist primarily of a cash-flow schedule, a set of keystrokes, and some conceptual reminders where necessary.

Before plunging headlong into this set of examples, you may want to take a few moments to go through the following knowledge checklist to make sure you haven't missed an important part of the discussion to this point.

Knowledge Checklist

❑ 1. You know all the basics: how to do arithmetic, how to use the M register for storing numbers, and how to store numbers to and recall numbers from the TVM registers. Plus, you know how to use [2ndF] [TAB] to adjust the number of displayed decimal places and how to use [+/-] to change the sign of the number in the display (page 6).

❑ 2. You know how to draw a cash-flow schedule for any financial situation that has a regular, even payment stream between the beginning and end of the contract. You know that you must maintain one perspective (that of a borrower or a lender) when you draw a cash-flow schedule (page 37).

❑ 3. You know the various parts of the cash-flow schedule and how they correspond to the five TVM keys on your EL-733A(page 39).

❑ 4. You know how to compute payments, present values, future values, and interest rates, and you have some feel for, or are becoming less intimidated by, the terminology that goes along with these values in various fields of finance.

Savings, IRA's and Annuities

As a quick example of the calculation of compound interest accumulation on a savings account, look at the following:

Example: How much money will you have at the end of 10 years if you put $10'000 in a savings acount and the interest rate is 9% APR compounded monthly? Assume you leave both the principal and interest in the account.

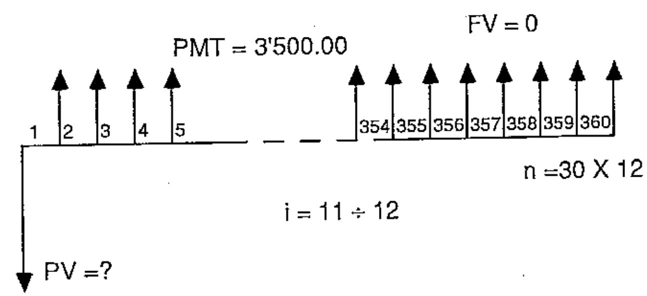
(Mode: FIN)

Keystrokes: 10'000 [+/-] [PV]
10 [2ndF] [X12] [n]
9 [2ndF] [÷12] [i]
0 [PMT]
[COMP] [FV]

Result: 24'513.57

Example: How much per month do you need to put away for the next twenty years if you wish to draw $3500 per month for the following thirty years? Assume a rate of return of 11% APR.

Solution: Work this problem backwards in two parts. The cash-flow schedule for the first part looks like this:



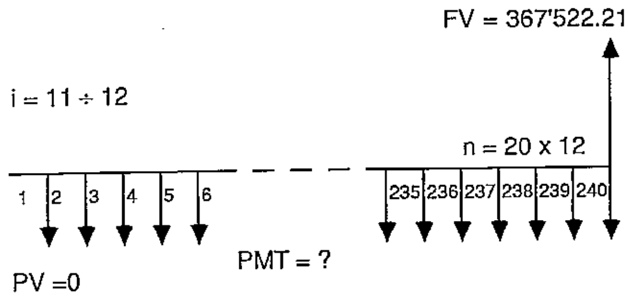
You need to calculate the amount that needs to be in an 11% account up front (the PV) if you wish to draw $3'500 per month for 30 years. This is a PV calculation like those on page 50. Make sure you are **not in BGN mode**:

(Mode: FIN)

3'500 [PMT]
30 [2ndF] [X12] [n]
11 [2ndF] [÷12] [i]
0 [FV]
[COMP] [PV]

Result: –367'522.21

The next step is to figure out what you need to put away each month for the next 20 years to arrive at the above amount ($367'522.21). This is the cash-flow schedule:



You don't have to change [i], but you have to change FV, PV, and n. With –367'522.21 sitting in your display, the keystrokes are as follows:

[+/-] [FV] (or [2ndF] [RCL] [PV] [+/-] [FV])
0 [PV]
20 [2ndF] [X12] [n]
[COMP] [PMT] Result: –424.57

It's always surprizing to compute how little it takes each month to build up a fairly sizeable annuity like this one.

Example: How much money do you need to deposit today in a college fund for your three-year-old child at 10.5% APR compounded monthly to ensure that child a college income of $1200.00 per month for four years, starting 15 years from today?

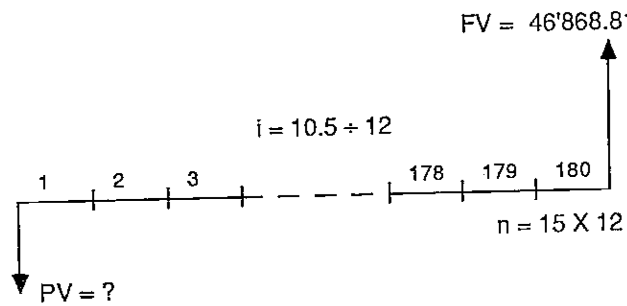
Solution: This is the same type of problem as the one above. The difference in calculation comes only in the second part: Rather than making

monthly payments, you are depositing one amount (a PV) and letting that grow without any additions. The keystrokes start out basically the same. Again, this is not in BGN mode:

(Mode: FIN)

1'200 [PMT]
4 [2ndF] [X12] [n]
10.5 [2ndF] [÷12] [i]
0 [FV]
[COMP] [PV] Result: –46'868.81

So by the time your child starts drawing on the college account 15 years from now, it must have accumulated a balance of $46'868.81. The second part is real simple. Only two arrows appear on the cash-flow schedule:



With –46'868.81 in the display, here are the keystrokes:

[+/-] [FV]
0 [PMT]
15 [2ndF] [X12] [n]
[COMP] [PV] Result: –9'768.89

The second part of this solution demonstrates the sliding of a cash-flow backwards in time. You slid an FV back to the front of the time line, adjusting it according to the interest rate regulating the schedule. Certain problems can only be solved using the TVM functions if you know how to slide cash-flows. Sliding cash-flows is discussed more starting on page 100.

Interest Rate Conversions ([→APR] and [→EFF])

Example: As one of the biggest printers on the west coast, you recently negotiated an eight-year lease on a $500'000.00 five color press. The terms of the lease call for quarterly payments in advance with a 20% residual. The quarterly payments are calculated on a 9.5% interest rate compounded monthly. What are the payments?

Solution: First compute the quarterly interest rate from the information given. (See page 72 if this gives you problems)

12 [2ndF] [→EFF] 9.5 [=] [X→M]
4 [2ndF] [→APR] [RM] [÷] 4 [=] Result: 2.39

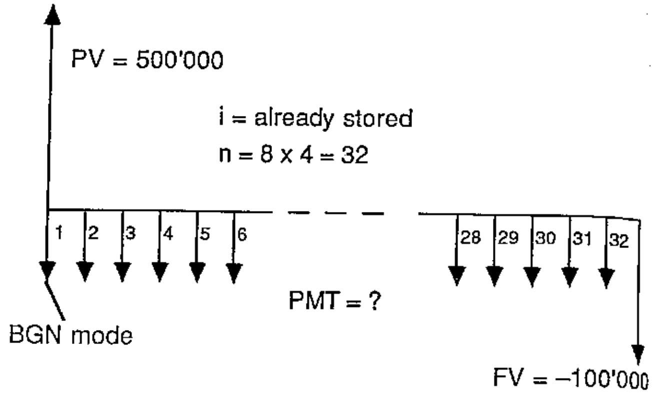
Once you have calculated the correct quarterly rate, store it as the interest rate. Press:

[i]

Then draw the cash-flow schedule. One term in the description that you may not be familiar with is "residual." A residual in a lease is a future value (FV) that is usually some percentage of the original value of the leased property. In this case the residual is 20%, which means 20% of $500'000 or $100'000.

Another thing to beware of when drawing the cash-flow schedule for this lease is "payments in advance." The payments occur at the beginning of the month, requiring your calculator to be set to **BGN mode** before you calculate the payment.

The cash-flow schedule that you come up with should look something like this:



As usual, once you have the correct picture drawn, the keystroke solution is easy:

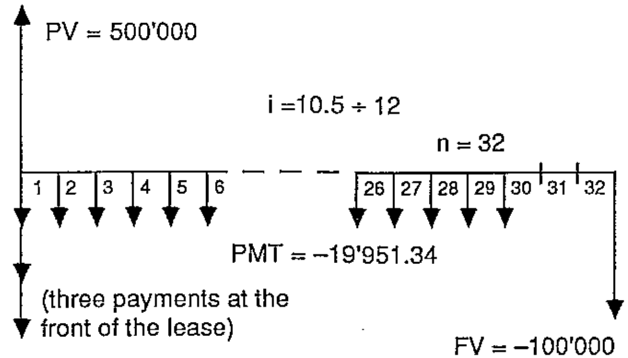
(Mode: FIN)

32 [n]
100'000 [+/-] [FV]
500'000 [PV]
[COMP] [PMT] Result: –19'951.34

If you got the result –20'428.94, you are not in BGN mode. Press [BGN] and recompute PMT.

Question: The leasing company requests the last two payments in advance, You agree to make those two payments in advance, but only if the overall interest rate does not exceed 10.5% APR compounded quarterly. (1) Would they have to reduce the price? (2) What would the overall interest rate be if you make the two advance payments and they do not lower the price?

Answer: The first part of the above question can still be answered using the TVM functions, but the cash-flow schedule changes enough that it no longer fits directly into the TVM registers:



The lease payment no longer runs all the way to the end of the time line, so you have to do a little before hand modifications before you can do a straight PV calculation. Since the interest rate is specified, you can slide that –100'000 value to the left on the cash-flow line so that the problem conforms to a TVM problem. Turn to page 100 if you wish to see how the first part of this question is solved using the TVM functions.

The second part of this question, where the interest rate is unknown, cannot be solved with the TVM functions. However, the second part of this question can be easily analyzed using the Discounted Cash-Flow Functions ([CFj], [Nj], [NPV], and [IRR]). Page 115 shows how it is solved using the Discounted Cash-Flow Functions.

Qualifying A Home Buyer

If you are in real estate sales, you know that banks use certain formulas to qualify a home buyer. These formulas vary slightly from state to state and from bank to bank, but the goal behind qualifying a buyer is to make sure that the buyer's income can support the burden of owning a home.

One rule of thumb is for qualifying a buyer is this:

If the buyer has current debt liability that is less than 35% of their gross income (income before taxes), then they can afford a payment (including taxes and insurance) of about 28% of their gross income.

Now, the taxes and insurance part of a mortgage payment varies considerably from state to state. In your state, a rule of thumb probably exists that says "taxes and insurance make up x-percent of a mortgage payment." In the state where this manual is being written, taxes and insurance consume about 25% (gulp!) of a mortgage payment. In your state it may be considerably different.

Example: A newlywed couple have an appointment with you in your real estate sales office. As you introduce yourself, you question the seriousness of these two bubbly characters, but your eyes light up when they ask you what price range of houses they can be looking at with a combined gross income of $4'200.00 a month.

In your state, the numbers to the rule of thumb for buyer qualification are as stated in the description that precedes this example. Interest rates are right at 10%. What is the price range of the houses that you can show this couple?

Solution: The first calculation is a percentage calculation. What mortgage payment can this couple qualify to make?

4'200 [X] 28 [%] Result: 1'176

The highest payment they can qualify for is $1'176.00. This assumes that they don't have more than 35% (or $1470 per month) tied up in other liablilities. But also, that $1'176 mortgage payment will include taxes and insurance, so you need to subtract those out before you do a PV calculation. Press:

[−] 25 [%] Result: 882.00

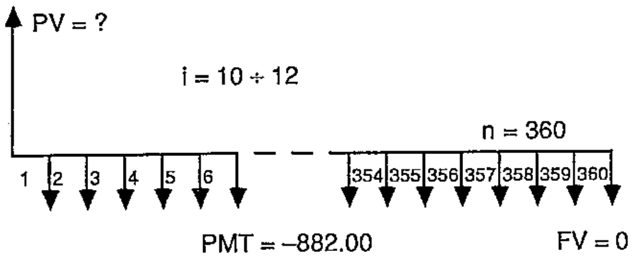
Remember, this 25% number for taxes and insurance varies from state to state.

Once you have subtracted the taxes and insurance from their mortgage payment store this ($882.00) as payment. Press:

(Mode: FIN)

[+/-] [PMT]

Then sketch out a cash-flow schedule (or visualize one in your mind) that represents a 30 year mortgage at 10% APR with a known payment and an unknown PV:



Finally, key in the three remaining numbers from that cash-flow schedule, and solve for the present value:

10 [2ndF] [÷12] [i]
30 [2ndF] [X12] [n]
0 [FV]

Make sure you are not in BGN mode!

[COMP] [PV] Result: 100'504.62

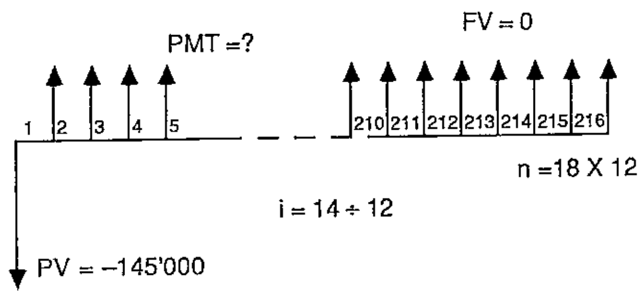
So the couple can probably finance about $100'500 on a house. They also have to meet a down payment requirement (around 10% down) before they can look for houses in this range.

Quoting An Actual Effective Interest Rate

Truth-in-lending laws require banks and financial institutions to quote actual, effective annual rates (including prepaid finance charges) and total interest paid on a loan or contract. The following example demonstrates how to quote an actual effective rate. Quoting total interest paid is demonstrated in the example that starts on page 94.

Example: The lending institution where you work loaned $145'000 to an individual for a term of 18 years. The nominal interest rate was 14% APR, compounded monthly, with a prepaid finance charge of 1.5%. Payments on the loan are monthly. What is the actual, effective APR on this contract?

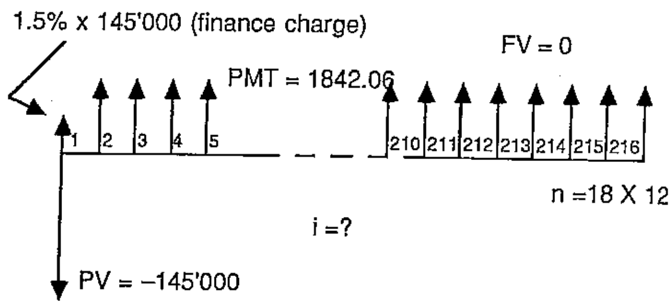
Solution: The first step in this solution is to calculate the payment based on the periodic rate (14 ÷ 12). The cash-flow schedule of the loan used to calculate this payment is as follows:



The keystrokes required to solve for the payment on this loan are (make sure that the FIN indicator is on in the display. If you don't know how to turn that indicator on and off, turn to page 10):

145'000 [+/-] [PV]
18 [2ndF] [X12] [n]
14 [2ndF] [÷12] [i]
0 [FV]
[COMP] [PMT] Result: 1842.06

Then, once you have calculated the payment, consider how the finance charges reduce the net amount of money loaned. This cash-flow schedule shows exactly what happens in this contract:



The up front finance charge reduces the net amount of money loaned without affecting the payment, and thus increases the actual interest rate:

[2ndF] [RCL] [PV] [–] 1.5 [%] [PV]
[COMP] [i] (pause)
[×] 12 [=] Result: 14.27

That is the actual annual percentage rate on this loan. However, this rate is still a straight APR computed by multiplying the periodic rate by the number of periods in a year. This rate is not an effective rate in that it does not incorporate compounding.

To compute this actual, effective rate, you need to use the [→EFF] key. First store that 14.27 in memory. Press:

[X→M]

Then compute the effective annual rate that results by compounding 14.27% twelve times:

12 [2ndF] [→EFF] [RM] [=] Result: 15.24

So the actual, effective APR quoted by your lending institution is 15.24%. That rate includes prepaid finance charges and compounding.

Amortization Schedules ([AMRT], [P/P], and [ACC])

An amortization schedule on a loan or mortgage separates, on a payment by payment basis, the amount of interest paid from the amount of principal paid. The EL-733A has three functions that allow you to create amortization schedules. These functions are:

[AMRT] to break a single payment into principal and interest,

[P/P] to enter a payment period for use by [ACC], and

[ACC] to show the accumulated interest and principal.

Using these three functions, you can create amortization schedules that quote principal and interest either payment by payment or over a series of payments.

The EL-733A uses information stored in the five TVM registers n, i, PV, FV, and PMT, when building an amortization schedule. Usually, you will compute the loan payment right before you create the amortization schedule, so the information in the TVM registers will already be correct. Look at this example:

Example: What is the payment on a 15-year mortgage of $69'000 at 9.65% APR compounded monthly? Also, what is breakdown of interest-vs-principal at the end of each of the first three years?

Solution: This type of payment calculation should be getting easy for you by this point in your reading. Here are the keystrokes:

(Mode: FIN)

69'000 [PV]
15 [2ndF] [X12] [n]
9.65 [2ndF] [÷12] [i]
0 [FV]
[COMP] [PMT] Result: –726.77

Now, since all the information about the loan is stored in the TVM registers, you can build the amortization schedule. To compute the principal, interest, and balance paid from the first payment to the twelfth payment, press:

1 [P/P] 12 [P/P] [ACC]

The display shows –2'156.51 and a display indicator "ΣPRN" comes on. ΣPRN stands for ACCUMULATED PRINCIPAL. This display tells you that the first twelve months of payments consist of $2'156.51 in principal.

Press [ACC] again. The display shows –6'564.77 and the indicator ΣINT comes on. This display tells you that the first twelve months of payments consist of $6'564.77 in interest.

For the next 12 months, press:

13 [P/P] 24 [P/P] [ACC] ΣPRN: –2'374.07
[ACC] ΣINT: –6'347.22

For the third 12 months, press:

25 [P/P] 36 [P/P] [ACC] ΣPRN: –2'613.57
[ACC] ΣINT: –6'107.71

You can continue breaking the loan down year by year in the above fashion until you reach the end of the 15th year.

Question: In the above loan, what is the interest and principal paid in the 45th payment, and what is the balance yet to be paid on the loan after that payment?

Solution: Press: 45 [AMRT] Result: –244.53

Notice the indicator "PRN" that came on in the display. This indicator tells you that the part of the 45th payment that is principal is now shown in the display. So the principal portion is $244.53.

Press [AMRT] again. This display shows –482.25 and the INT indicator comes on in the display. This says that the portion of the 45th payment that is interest is $482.25.

Finally, to see the balance of the loan after the 45th payment, press [AMRT] once again. The number 59'724.05 shows in the display along with the indicator "BAL." The balance of the loan after the 45th payment is $59'724.05.

Quoting Total Interest Paid On A Contract

The [ACC] function allows you to quote the total interest paid on a TVM problem with the push of a button. Work the above example on creating an amortization schedule to get an idea of how the [ACC] function works. Then, to calculate the total interest paid on the loan in the above example, just press:

1 [P/P] 180 [P/P] [ACC]

This calculation will take a long time (around 30 seconds), but when it is finished, the display will show –69'000.00 with the ΣPRN indicator on in the display to tell you that the entire principal of the loan has been paid off.

Press [ACC] again to see the total interest paid during this loan. The ΣINT indicator comes on and the display shows –61'819.22. From the first payment to the last payment of this loan, a total of $61'819.22 in interest is paid.

In summary, quoting the total interest paid on a loan or contract is a matter of computing the accumulated interest over the entire term of the contract using the [ACC] key.

Discounted Loans And Mortgages

Occasionally, in order to become more liquid, a lender will try to sell a contract for a price that is discounted from the actual balance left to pay on the contract. The resulting yield to the party that purchases the contract is higher than the original rate that the contract was based on.

The word "discounted" is used here to mean reducing the value or price of a contract in order to sell it. The word "discounted" is applied differently in "Discounted Cash-Flow Analysis" where it refers to a process of solving

financial problems by sliding or "discounting" cash-flows on a cash-flow schedule.

If you are ever considering buying a discounted contract and you wish to calculate what you will yield or the price you should pay for the contract to yield a certain rate, don't make the mistake of getting cluttered up with the numerical details of the original contract. The trick to making accurate calculations on a discounted contract is to look only at the payment schedule that you are buying and to do your calculations from that.

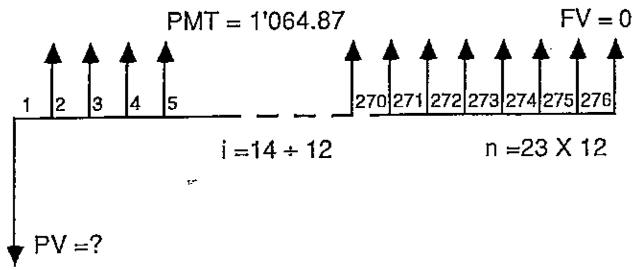
If the payment schedule is regular and even, you can calculate the yield or the amount to pay for the contract with the TVM functions. If the payment schedule is uneven or irregular, you will probably need to use the Discounted Cash-Flow Analysis functions. Look at this example of a discounted mortgage problem that can be solved using the TVM functions.

Example: A lender wishes to sell a mortgage that started about seven years ago. The mortgage was originally written for $115'000 at 10.65% for 30 years. A 2% prepaid finance fee was paid at the beginning of the loan and the house for which the mortgage was written was selling for $135'000 requiring the borrower to come up with a down payment of $20'000. The house is now valued at about $200'000. The payments on the mortgage up to this point have all been on time, and the remaining payment schedule calls for 23 years of monthly $1'064.87 payments, starting at the end of the current month. What should you pay for this mortgage if you wish to cash out the lender and yield 14%?

Solution: The above paragraph contains a lot of wasted words. But so do most real life situations (and test problems often have redundant information thrown in to lead you off track).

You are looking for an investment. The above investment offers you a monthly income of $1'064.87 for the next 23 years. That is all you need to know in order to do the calculation! The information about the house and the original terms of the contract is unnecessary (other than for you to use in weighing the security of the investment).

You have to pay some money up front in order to gain the right to receive that monthly income of $1064.87. You want that amount to be such that your investment yields 14% APR. The cash-flow schedule looks like this:



So all those words boil down to a simple PV calculation. By storing your desired yield of 14% as the interest rate, the calculated PV is the amount you need to pay in order to yield 14% on the investment.

The keystrokes are as follows:

(Mode: FIN)

14 [2ndF] [÷12] [i]
1064.87 [PMT]
23 [2ndF] [X12] [n]
0 [FV]
[COMP] [PV] Result: –87'559.08

In order to yield 14% APR compounded monthly by purchasing the contract, buy it for $87'559.08.

Question: If the asking price is $95'000, what will be your yield?

Answer: Change the PV and recalculate the periodic interest rate. Once you have the periodic interest rate, annualize it by multiplying by 12.

95'000 [+/-] [PV]
[COMP] [i] (pause)
[X] 12 [=] Result: 12.72

Is that close enough to your desired return? If not, make them an offer.

Sliding Cash-Flows

By the time you reach this section in the manual, you should recognize that both the EL-733A and the cash-flow schedule are dynamic tools for use in financial calculations. This section demonstrates sliding cash-flows on the cash-flow schedule to simplify certain problems and to make certain complicated problems solvable.

We'll start by stating the following:

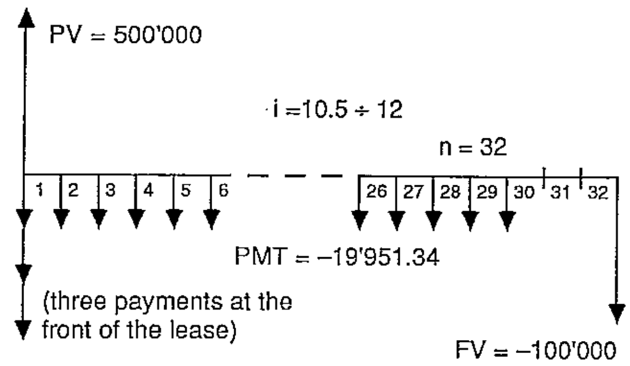
You can slide any cash-flow on a cash-flow schedule to any period on that schedule without changing the results of a calculation, **as long as you adjust that cash-flow according to the known interest rate that prevails on the cash-flow schedule.**

From the above statement, you can see that in order to slide a cash-flow to another period on a cash-flow schedule, **you have to know** the prevailing interest rate.

Though you may have not realized it at the time, you were using the skill of sliding cash-flows in the annuity problem on page 82. The mechanics of sliding cash-flows using the TVM functions are the same as the second part of that problem.

The first part of the question at the end of the lease problem that starts back on page 85, is a good example to use to demonstrate answering a question by sliding a single cash-flow using the TVM registers.

The cash-flow schedule of that lease problem with advance payments looks like this:



The last two payments are made at the beginning of the lease, and the lease was written with the payments in advance, so a total of three payments are made at the time that the deal is closed. You agreed to make those payments in advance, as long as the overall interest rate did not exceed 10.5% APR compounded quarterly. (Remember, the payments were quarterly but were calculated based on a 9.5% APR compounded monthly.)

In order to solve for the PV, you must first slide that -100'000 FV back to either the end of the 29th or end of the 30th period (adjusting for the 10.5% interest rate). We chose to slide it to the end of the 29th period so that the final solution would not require BGN mode.

To slide a cash-flow to the left using the TVM functions, you enter that cash-flow as an FV:

100'000 [+/-] [FV] .

Key in the interest rate.

10.5 [÷] 4 [=] [i] Result: 2.63

Key in the number of periods you wish to slide it:

3 [n]

Key in 0 for the payment:

0 [PMT]

And compute PV:

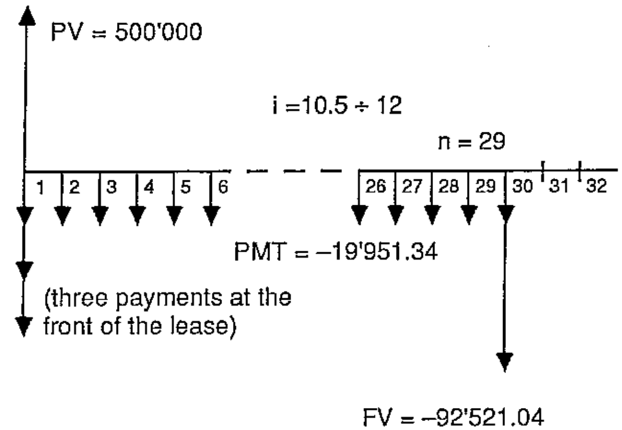
[COMP] [PV] Result: 92'521.04

This value comes out positive, but you will have to store it as a negative. The TVM functions always assume that the FV is a return on a PV invested at some earlier time. When you are sliding cash-flows this assumption is not correct, so *you* have to keep track of the correct sign (positive or negative). Press:

[+/-] [FV]

(To slide a cash-flow to the right, you use the above process, except you enter the cash-flow as PV and compute FV to do the sliding.)

Now you can draw a cash-flow schedule that fits into the TVM registers. It should look like this:



This cash-flow schedule shows the three payments that occur at the front of the lease so that you remember them when you are interpreting the result of this PV calculation.

Both [i] and [FV] have already been stored:

19'951.34 [+/-] [PMT]
29 [n]

(make sure that BGN is not on in the display)

[COMP] [PV] Result: 445'182.80

That PV includes the three payments at the front of the lease, so to see what the actual lease value is, add those payments in:

[X÷M]

2ndF RCL PMT +/- × 3 =
M+ RM

Result: 505'036.82

But the lease value for the equipment was only $500'000. The demand for the payments up front has the same effect as increasing the price $5'036.82 and increasing the interest rate to 10.5% APR compounded quarterly. Another way to look at it is that it has the same effect as a more drastic increase in rates (the calculation of this rate is shown on page 115). So, unless they lower the price by over $5'000.00, this deal does not meet your demand for a 10.5% maximum interest rate.

Discounted Cash-Flow Analysis

The previous example leads quite well into the topic of this section. The two functions NPV and IRR on the EL-733A depend (numerically) on the fact that cash-flows can be moved up and down a cash-flow schedule. The process that you just used to slide back that one cash-flow is similar to the calculating process that these functions depend upon.

In discounted cash-flow analysis, the parts of the cash-flow schedule take on new meanings. In TVM analysis, you had n (the number of periods), i (the periodic interest rate), PV (the cash-flow at the beginning of the first period), FV (the cash-flow at the end of the last period), and PMT (a regular stream of level cash-flows from the PV to the FV), allowing you to describe many financial situations to your EL-733A. But a regular, level stream of payments can be a big restriction, and, depending on how creative your field of finance is, it may be necessary to generalize your perception of the cash-flow schedule to incorporate the exciting capabilities of discounted cash-flow analysis.

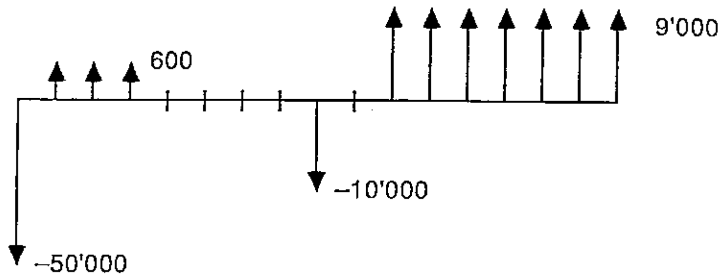
With discounted cash-flow analysis, you need to be able to describe any cash-flow schedule using just two functions:

2ndF Ni : The number of cash-flows in cash-flow group i.

CFi : The value of the cash-flows in cash-flow group i.

You need to throw out the TVM values of n, PV, FV, and PMT, and think in terms of cash-flow groups. Every cash-flow schedule is made up of connected groups of cash-flows. These groups are characterized by the value of each cash-flow making up the group and the number of cash-flows in the group. A cash-flow group could consist of ten, $100 cash-flows; fifty, $10'000 cash-flows; or up to ninety-nine cash-flows of the same amount all in a row.

These cash-flow groups are numbered from left to right (starting at zero!) on the cash-flow schedule. Here is an example of a cash-flow schedule with 6 cash-flow groups (numbered 0 to 5).



- Cash-flow group zero has **one** cash-flow of **–50'000.**
- Cash-flow group one has **three** cash-flows of **600** each.
- Cash-flow group two has **four** cash-flows of **0** each.
- Cash-flow group three has **one** cash-flow of **–10'000.**
- Cash-flow group four has **one** cash-flow of **0.**
- Cash-flow group five has **seven** cash-flows of **9'000** each.

Cash-flow group zero is always the initial group on a cash-flow schedule. Most often, cash-flow group zero will consist of just one cash-flow, but it can consist of up to 99. It is called "cash-flow group zero" because the first cash-flow in that group starts at the *beginning* of the first period, not the end. (Also, zero corresponds to the numbered register in which it is held, but that is not discussed until page 117.)

Once you can think of a cash-flow schedule in terms of groups of cash-flows, you will have no problem using the two functions [CFi] and [Ni] to describe any realistic cash-flow schedule to your EL-733A.

Example: Describe that cash-flow schedule with six groups of cash-flows to your EL-733A.

Solution: First, make sure your EL-733A is in FIN mode, then press [2ndF] [CA] to "clear all" of the financial registers. To input the cash-flow group zero, press:

1 [2ndF] [Ni]
50'000 [+/-] [CFi]

To key in the cash-flow group one, press:

3 [2ndF] [Ni]
600 [CFi]

Then continue this process for the cash-flow groups two through five:

4 [2ndF] [Ni]
0 [CFi]

1 [2ndF] [Ni]
10'000 [+/-] [CFi]

0 [CFi]

7 [2ndF] [Ni]
9'000 [CFi]

Notice that on cash-flow group four, which consisted of one cash-flow of 0, we left off the keystrokes [1] [2ndF] [Ni]. You can always leave off the keystrokes [1] [2ndF] [Ni]. We included them in the previous groups consisting of just one cash-flow for clarity. But whenever you press [CFi] without first pressing [2ndF] [Ni], the EL-733A assumes that there is just one cash-flow in that group.

After completing the above solution, you have described a cash-flow schedule with very irregular cash-flows to your calculator. Notice that in describing this cash-flow schedule, you have accounted for the beginning of the first period and the end of **every period**. You have not left off the periods that have cash-flows of zero.

So, with the information that you have keyed in, the calculator can deduce exactly what happens at each period on the cash-flow schedule, from the beginning of the first period to the end of the last period. That cash-flow schedule is now stored in the memory of your EL-733A, and the EL-733A is ready to answer either one of these two very important questions about that schedule:

1. Given a periodic interest rate (stored in the i register), what is the value of all the cash-flows on that schedule if they are slid to the beginning of the first period (discounted according to the given interest rate) and netted together? In other words, given a periodic interest rate, what is the Net Present Value ([NPV]) of the cash-flows on that schedule?

2. What is the periodic interest rate that would make the Net Present Value equal to zero? This interest rate is called the Internal Rate of Return (or [IRR]).

The answers to those two questions open up a literal wealth of information about the majority of financial problems with irregular cash-flows. The remainder of this chapter looks at how you can apply the answers to those two questions to the financial scenarios that you encounter.

[NPV] : NET PRESENT VALUE

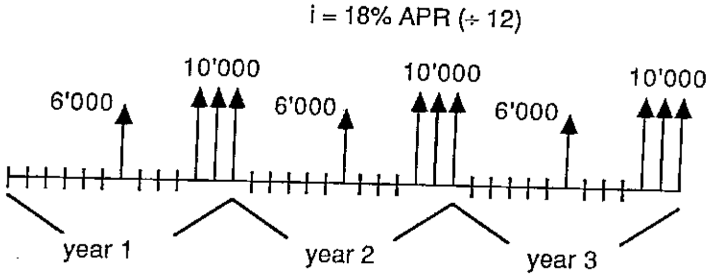
To illustrate the use of the [NPV] function, let's look at an an example. This is similar to the discounted mortgage example on page 96, except that the payment schedule is not regular.

Example: At a New Year's Eve party, you are approached by a lender who wishes to sell a contract. Though not anxious to discuss business during the festivities, you are intrigued by the potential good deal that you are being offered, so you take your EL-733A from your pocket to do some quick calculations.

The payment schedule on the contract this lender is selling calls for a $6'000 payment at the end of June and a $10'000 payment at the end of each of the three months October, November, and December for the next three years. You have some cash in a mutual fund that has been getting about a 12% return, and you would like to boost that return to around 18%. What should you pay for the contract?

Solution: This is a typical situation where you can make good use of the NPV function. You know (or can specify) the periodic interest rate, and you are interested in what the schedule is worth up front.

This is what your sketch of the cash-flow schedule should look like:



The part of any NPV or IRR calculation that takes the most keystrokes is describing the cash-flow schedule to your calculator. An important thing to notice before you start keying in the above cash-flow schedule is that the group at the beginning of the first year (cash-flow group zero) consists of six cash-flows of zero each, while the groups at the beginning of the second and third years have only five cash-flows of zero each.

The keystrokes necessary to key in the above cash-flow schedule are as follows:

(Mode: FIN)

[2ndF] [CA]

6 [2ndF] [Ni] 0 [CFi]	Cash-flow group 0
6'000 [CFi]	Cash-flow group 1
3 [2ndF] [Ni] 0 [CFi]	Cash-flow group 2
3 [2ndF] [Ni] 10'000 [CFi]	Cash-flow group 3
5 [2ndF] [Ni] 0 [CFi]	Cash-flow group 4
6'000 [CFi]	Cash-flow group 5
3 [2ndF] [Ni] 0 [CFi]	Cash-flow group 6
3 [2ndF] [Ni] 10'000 [CFi]	Cash-flow group 7
5 [2ndF] [Ni] 0 [CFi]	Cash-flow group 8
6'000 [CFi]	Cash-flow group 9
3 [2ndF] [Ni] 0 [CFi]	Cash-flow group 10
3 [2ndF] [Ni] 10'000 [CFi]	Cash-flow group 11

Once you have the cash-flow schedule stored in your EL-733A, to calculate the NPV given an interest rate of 18%, simply press:

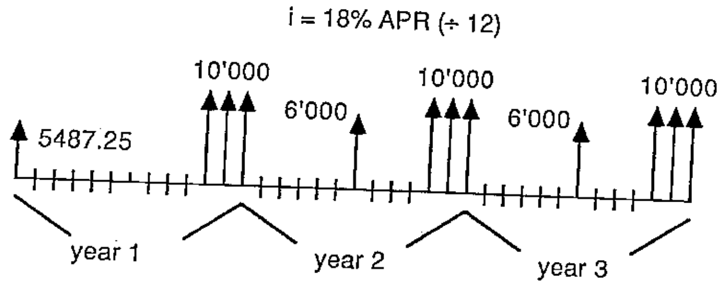
18 [2ndF] [÷12] [i] [NPV]   Result: 78'505.16

So, if you want to make an 18% APR return on your investment by buying that contract at the New Year's Eve party, you should pay $78'505.16

Notice that the result of sliding all those positive cash-flows to the beginning of the time-line and discounting them according to the 1.5% periodic rate, is a positive value. Unlike the TVM functions [PV] and [FV], the [NPV] function makes no assumptions as to why you are using it to slide cash-flows. [NPV] does not change the signs of cash-flows as it slides them to the front of the cash-flow schedule.

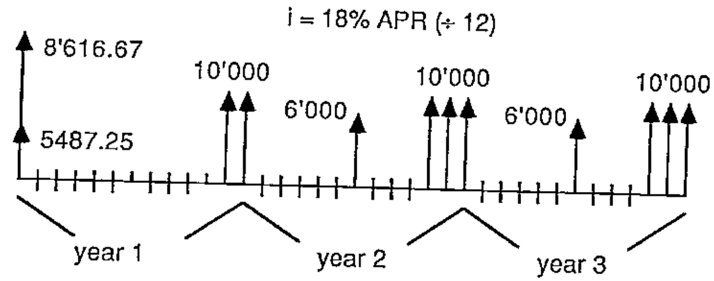
(For more details on sliding cash-flows with the TVM functions, you may want to review page 100.)

You can think of NPV as though it treats each cash-flow as a separate loan. In the previous example, if you slide just the first $6'000 cash-flow to the front of the time line, adjusting it for the interest rate, the cash-flow schedule would look like this:

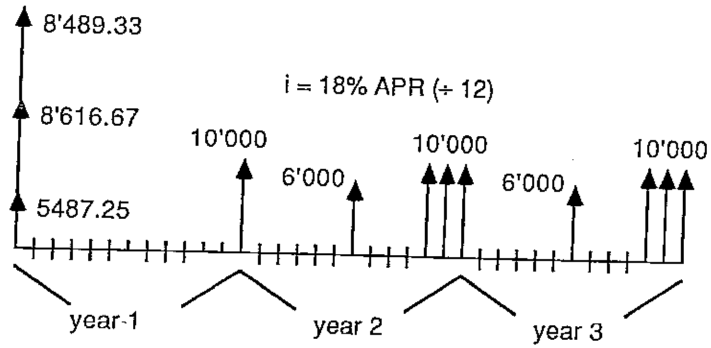


By sliding the first $6'000 cash-flow back six periods and adjusting for the 1.5% periodic interest rate, you reduce its value (you discount it) to about $5487.25.

Next if you slide the first $10'000 cash-flow to the front of the time line, it looks like this:

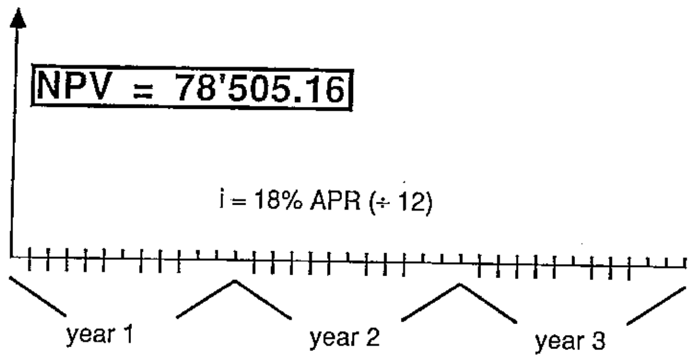


Then, slide the next $10'000 cash-flow to the front:



Notice that, because it is slid back one period farther than the first $10'000 cash flow, the second $10'000 cash-flow is worth less up front.

Continue this sliding process until every cash-flow has been "discounted." Finally, net them together to arrive at the Net Present Value of $78'505.16:



The Net Present Value of a cash-flow schedule tells you the present value of a stream of cash-flows based on a known interest rate.

What is the Net Present Value of, for example, any complete picture of a mortgage? Well, if you include the PV, FV (if there is one), and PMT's all on the same picture, then at the interest rate used to calculate the payment, the NPV will be zero. **The NPV of any complete picture of an amortized investment is zero.**

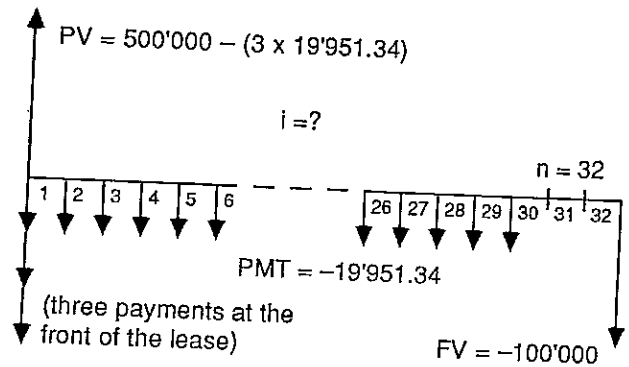
In the example in this section, if you had keyed in a cash-flow of –78'505.16 as the first cash-flow on the cash-flow schedule, the Net Present Value at the desired 18% interest rate would be zero. With the –78'505.16 up front, this interest rate would be said to be the Internal Rate of Return (IRR).

[IRR] INTERNAL RATE OF RETURN

The Internal Rate of Return is the periodic interest rate that causes the Net Present Value of a cash-flow schedule to be zero. The IRR function is used most often to tell you the return on an investment with an uneven or irregular payment stream.

Whenever you calculate a result by pressing the [IRR] key on the EL-733A, the result is stored in the i register. You can then use this result in a TVM problem, or you can recall it by pressing [2ndF] [RCL] [i].

In the lease problem on page 85, the second part of the question at the end of the solution asked you to calculate the interest rate on this cash-flow schedule (the lease with payments in advance):



The only way to solve for the interest rate on a cash-flow schedule like this one that has uneven cash-flows is to use the [CFi], [Ni], and [IRR] functions. The above cash-flow schedule represents a complete investment (in other words, every cash-flow that applies to this investment is shown). So once you key in this cash-flow schedule, the calculation of the IRR is a one key operation. Keying in cash-flow schedules like this is described in the previous section starting on page 105. Here are the keystrokes:

(Mode: FIN)

[2ndF] [CA]

440'145.98 [CFi]

29 [2ndF] [Ni] 19'951.34 [+/-] [CFi]

2 [2ndF] [Ni] 0 [CFi]

100'000 [+/-] [CFi]

After pressing the above keys, the cash-flow schedule for the lease with advance payments is stored in your EL-733A. To calculate the quarterly rate that governs that schedule, press:

[IRR] Result: 2.70

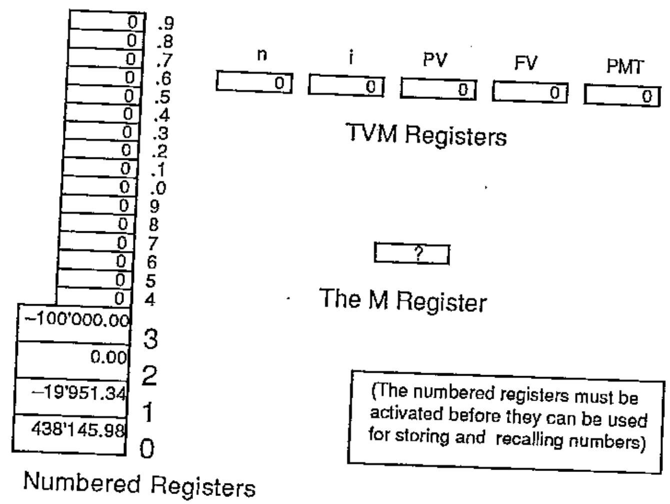
This calculation may take a little time. Multiply the above result by four to get the annualized rate, which is 10.81%. So as you can see, by making payments in advance you are effectively increasing the interest rate you pay.

NOTICE that, when keying in your numbers, you should do all calculations ahead of time. You should not do any calculations while you are in the middle of keying in a list of [CFi]'s and [Ni]'s. More examples of NPV and IRR calculations start on page 119.

WHERE DO ALL THOSE NUMBERS GO?

Before you look at more examples in Discounted Cash-Flow Analysis, have you been wondering where all those numbers (cash-flows and n's) go when you key them in? They are all stored in the memory registers of your calculator, and you can change them individually if you so desire.

If you have just worked through the preceding example, the memory of your calculator has numbers stored in it as shown on the following diagram:



The numbered registers 0 through 4 are shown enlarged so you can read their contents. These registers contain the values of the cash-flow groups on the last cash-flow schedule that you keyed in.

To view any of these cash-flows, simply press [2ndF] [RCL] followed by the number of the register in which the cash-flow is stored. You can also change the values (N*i* and CF*i*) of a cash-flow group using the keystrokes shown in the upcoming examples. Notice that when you reach 9, the numbers start over with .0 to .9 so you can input up to 20 cash-flow groups.

But what about the values for N*i*? Where are they stored? Well, there are actually an additional 20 hidden special registers where the values for N*i* are stored. You can check or change any of the N*i* values that you input. **The hidden N*i* registers are short registers that can only hold numbers up to 99. This means that the largest number of cash-flows you can have in one group is 99.**

Example: Recall the amount in cash-flow group one from that last lease problem. Also, recall N_1.

Solution: Press [2ndF] [RCL] [1] to see the amount in cash-flow group one (–19'951.34). **After you have recalled the amount in that cash-flow group,** you can then check the number of cash-flows in that group by pressing [2ndF] [RCL] [2ndF] [Ni]. You will see the number 29.00 come into the display.

In order to recall a value in an N*i* register, you have to first recall the cash-flow amount from the numbered register that goes along with that N*i* register.

CHANGING N*i* AND CF*i*

Example: Change the amount in cash-flow group one to –21'000 and change N*i* to 35.

Solution: Press 35 [2ndF] [Ni] 21'000 [+/-] [2ndF] [STO] [1] to change N*i* and CF*i* in cash-flow group one.

To change either of the values in a cash-flow group, simply enter the new Ni and press [2ndF] [Ni] then enter the cash-flow amount and press [2ndF] [STO] followed by the number of the cash-flow group. Whenever you change either N*i* or CF*i* in a cash-flow group, you have to re-enter both numbers as shown above.

With these new numbers, you can recalculate IRR by pressing [IRR]. The result is 3.60.

After reading to this point from page 105, you have covered a good deal of background information on Discounted Cash-Flow analysis. The examples in the section that follow give you plenty of practice solving Discounted Cash-Flow Analysis problems.

DISCOUNTED CASH-FLOW EXAMPLES

Financial calculations that require Discounted Cash-Flow Analysis cover quite a range of investment situations. Often, the problems are fairly complex and require some of your time to be spent up front preparing a cash-flow schedule. Many times, in order to reach the final cash-flow schedule for a Discounted Cash-Flow Analysis problem, several TVM calculations may be required.

The first example in this section is a fairly straight-forward general investment analysis problem.

GENERAL INVESTMENT ANALYSIS

Example: Over the years you have invested in mutual funds. Your yield in these funds has been good, though you have never sat down and figured out exactly what that yield has been. One evening you find yourself with some time on your hands, so you pull out your records, take your EL-733A from your briefcase and start calculating. Based on the following table of your investments, what has your return on mutual funds been since 1983?

Date	Fund #	Amount Invested
11/15/83	1	5'000
6/1/84	1	3'200
8/15/85	2	2'000
4/30/86	3	2'000
1/15/87	2	2'750
6/1/87	3	1'500
9/1/87	2	4'000

Today is 5/1/88 and the values of the funds as of this date are as follows:

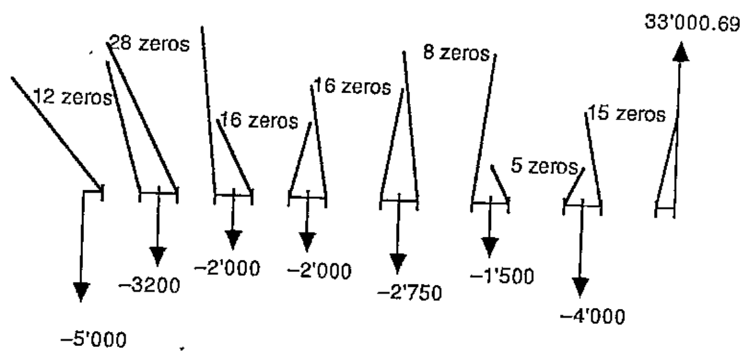
- Fund 1: $15'355.70
- Fund 2: $12'921.24
- Fund 3: $4'723.75

Solution: Even though there are three funds listed with three separate yields, in this case you are interested in the overall, combined yield. This becomes just one problem with all of the investments and current values appearing on one cash-flow schedule.

One trick to analyzing this investment is in the way that you choose the period of the cash-flow schedule. Here, the investments took place either around the 1st of a month or around the 15th of a month. So, break each year up into 24 periods, each about 1/2 month. That way, each investment can fall at the end of a period.

Once you have calculated the periodic return for 1/2 month periods, you can convert it to reflect monthly compounding if you wish.

The cash-flow schedule that you arrive at for this problem will look something like this:



This is a difficult one to draw clearly on one line.

On this cash-flow schedule, only the investments and the final value of the funds are represented with arrows. The periods that passed with no investment made are shown as "zeros, " because you will key them into your calculator as zeros. Here are the keystrokes to describe that cash-flow schedule to your EL-733A.

[2ndF] [CA]	
5'000 [+/-] [CFi]	Group 0
12 [2ndF] [Ni] [OCE] [CFi]	Group 1 (12 zeros)
3'200 [+/-] [CFi]	Group 2
28 [2ndF] [Ni] [OCE] [CFi]	Group 3 (28 zeros)
2'000 [+/-] [CFi]	Group 4
16 [2ndF] [Ni] [OCE] [CFi]	Group 5 (16 zeros)
2'000 [+/-] [CFi]	Group 6
16 [2ndF] [Ni] [OCE] [CFi]	Group 7 (16 zeros)
2'750 [+/-] [CFi]	Group 8
8 [2ndF] [Ni] [OCE] [CFi]	Group 9 (8 zeros)
1'500 [+/-] [CFi]	Group .0
5 [2ndF] [Ni] [OCE] [CFi]	Group .1 (5 zeros)
4'000 [+/-] [CFi]	Group .2
15 [2ndF] [Ni] [OCE] [CFi]	Group .3 (15 zeros)
33'000.69 [CFi]	Group .4

Finally, calculate your return by pressing:

[IRR] Result: 0.73

IRR calculations can take time on your EL-733A, depending on how many cash-flow groups are entered. The calculator is approaching the answer through a numerical guessing process. Many guesses are required before it arrives at a solution.

The final result is a periodic rate based on 24 half-month periods. To annualize this rate, multiply by 24 and you will get 17.55%. That is not a bad rate of return.

You can convert this 17.55% rate from an annual rate compounded semi-monthly to an annual rate compounded monthly using the following keystrokes (for more information on interest rate conversions, see page 69):

[X→M]
24 [2ndF] [→EFF] [RM] [=]
[X→M]
12 [2ndF] [→APR] [RM] [=] Result: 17.61

Again, that is not a bad return at all. Happy investing!

WRAP-AROUND MORTGAGES

A wrap-around mortgage is a second mortgage on a piece of real property that absorbs the first mortgage. The lender in a wrap-around mortgage usually agrees to assume the payments of the first mortgage and lend an additional sum beyond the Present Value of the first mortgage, in exchange for a periodic payment that is either greater than the payment on the first mortgage or that continues for a longer term.

The questions that you need to be able to answer to perform a wrap-around mortgage calculation are three in number:

1. What is the Present Value (remaining balance left to pay) on the first mortgage? Usually this is a TVM calculation.

2. What is the payment that the borrower must make on the new wrap-around mortgage? Usually this is another TVM calculation.

3. What is the yield to the lender on the whole scenario? (This is an IRR calculation)

So every wrap-around mortgage problem is at least three problems, involving three cash-flow schedules, the last of which is generally an uneven cash-flow schedule that needs to be solved using IRR.

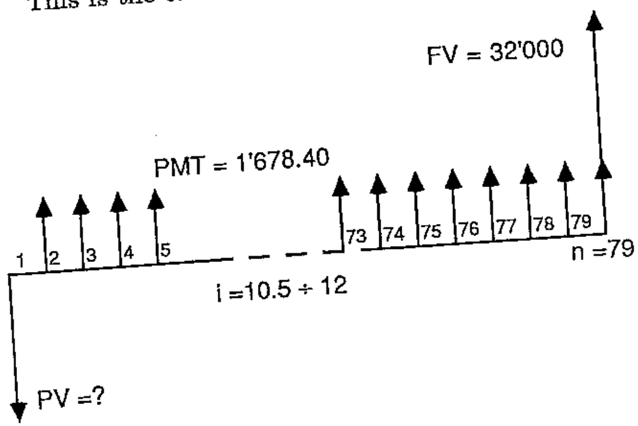
What is interesting about wrap-around mortgages is that the lender usually gets a better return on investment than the interest rate on either the first mortgage or the wrap-around. The payment on the first mortgage may have been calculated based on a 10.5% interest rate, the payment on the wrap may be calculated on a 13.5% rate and the overall rate of return may be 15%. It may seem like that is not possible, but...

Example: As a lender looking for a fantastic long term investment, you are approached by a person who wishes to borrow against some property and make just one monthly payment. This potential borrower has a single mortgage, written at 10.5% APR, that calls for $1'678.40 end-of-the-month payments with a $32'000 balloon payment immediately after the last payment, which is 79 months from today. The person would like to borrow an additional $50'000.

You agree to write a wrap-around mortgage at 12% for 30 years that has a $15'000 balloon. You will assume the payments of the first mortgage and lend this person an additional $50'000. In exchange, the borrower will make regular monthly payments to you for the next 30 years.

What is the payment that you will receive from the borrower each month, and what are you yielding by agreeing to wrap the mortgage?

Solution: The first step is to calculate the Present Value of the first mortgage. You are given the payment schedule, and all the necessary knowns, so this step is very much like the problem on page 50. This is the cash-flow schedule:



The keystrokes for this PV calculation are:

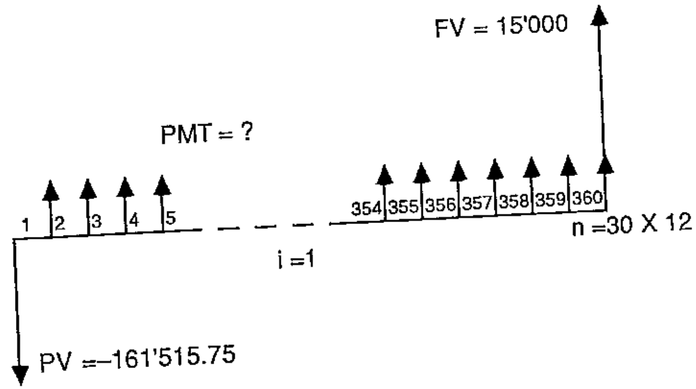
(Mode: FIN)

[2ndF] [CA]
1'678.4 [PMT]
79 [n]
10.5 [2ndF] [÷12] [i]
32'000 [FV]
[COMP] [PV]

Result: –111'515.75

So the first loan is worth $111'515.75 based on the original 10.5% APR.

Next, you need to calculate the payment on the new loan. You are loaning an additional $50'000 to this person, so add that to the above number and store it as the PV. The cash-flow schedule to calculate the payment on this wrap-around is as follows:



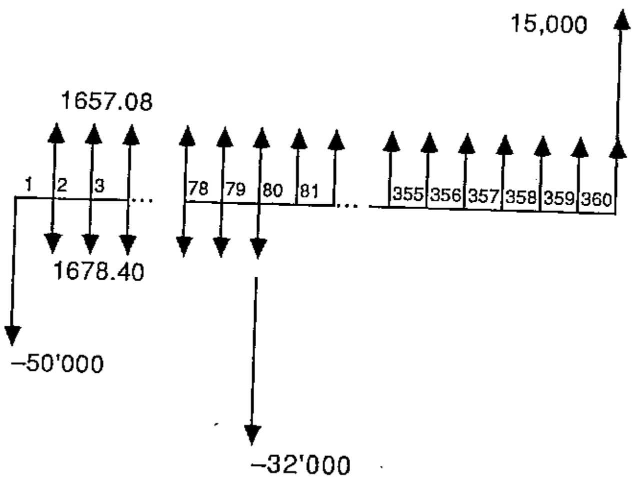
This, again is a simple TVM calculation. Assuming you have been following along, the keystrokes are:

[2ndF] [RCL] [PV] [–] 50'000 [=] [PV]
15'000 [FV]
1 [i]
30 [2ndF] [X12] [n]
[COMP] [PMT]

Result: 1'657.08

The borrower will be happy to see that this monthly payment is actually lower than the current mortgage payment. Granted, it continues for 360 months instead of 79 months, but that may be beside the point...?

OK. Now you are equipped with all the information that you need to calculate the return that you will be getting by wrapping this mortgage. Here is what this wrap-around mortgage looks like from your perspective:



You have agreed to loan $50'000 up front, to make seventy-nine $1'678.40 payments, and to make one 32'000 balloon payment at the end of the 79th month.

In exchange, the borrower has agreed to make monthly $1'657.08 payments to you for the next 360 months and to pay you a balloon of $15'000 at the end of those 360 payments.

That may look long and complicated, but it really consists of only seven cash-flow groups.

Remember when you are keying in the above cash-flow schedule, the maximum number you can key in for *Ni* is 99. This means that you will have to break that last stream of payments into three groups.

Again, assuming you have been following this example from the beginning, the keystrokes to key in the above schedule are as follows:

[2ndF] [RCL] [PMT] [X-M]
[2ndF] [CA]

50'000 [+/-] [CFi]	Group 1
78 [2ndF] [Ni] 20.74 [+/-] [CFi]	Group 2
32'020.74 [+/-] [CFi]	Group 3
82 [2ndF] [Ni] [RM] [CFi]	Group 4
99 [2ndF] [Ni] [RM] [CFi]	Group 5
99 [2ndF] [Ni] [RM] [CFi]	Group 6
16'657.08 [CFi]	Group 7

Then, once you have it all keyed in, calculate your rate of return by pressing:

[IRR] Result: 1.06

This calculation may take a little while, so be patient. This is a good time to stretch your legs and get a cup of tea. If you stay away too long and the calculator turns itself off, when you come back just remember that the result is in the i register and you can recall it by pressing [2ndF] [RCL] [i]. Annualize this result by multiplying by 12 to get a result of 12.69% APR.

Notice that the return here is greater than the interest rate you gave the borrower. Why is this?

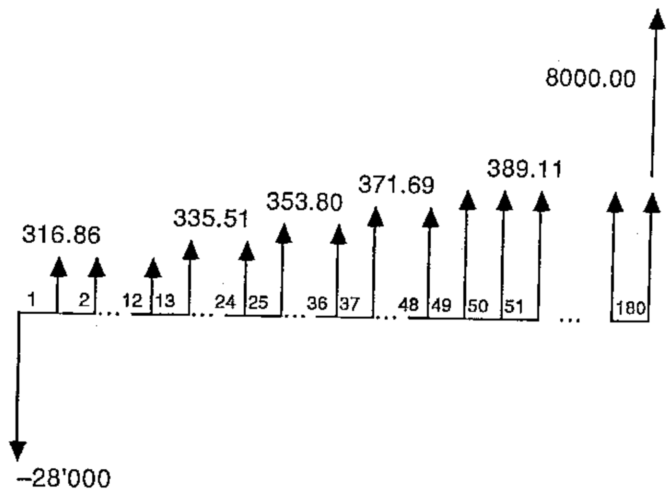
Well, with a wrap-around mortgage, you are earning interest on money you never had in the first place. In the extreme, it is like someone coming up to you and saying, "I'll pay you monthly payments that amount to 12% on this money that I borrowed at 10.5% if you will turn around and make my monthly 10.5% payments."

You don't need to come up with a huge amount of cash up front, you just need to be able to make the payments on someone else's loan (mostly with money that they are paying you). You are earning 1.5% (12 – 10.5) on money that someone else borrowed somewhere else. The larger the sum is that you wrap, the more money you make. Hmmm...

BLENDED RATE CONTRACTS AND MORTGAGES

If you ever come in contact with a variable rate loan or mortgage, you may be interested in the blended rate. The blended rate is just the IRR on the loan if you take into account the whole picture. As stated on page 74, with variable rate loans, the only scenario that makes sense to look at beforehand is the "worst case scenario." Once the payment schedule for the worst case scenario is calculated, then you can look at that schedule to see what the blended rate is over the term of the loan.

Example: Not long ago, you calculated the payment schedule for a variable rate loan. That payment schedule is described on page 76 and the resulting cash-flow schedule follows. Key in this schedule and calculate the blended rate by using [IRR].



Solution: The keystrokes used to describe this schedule to your EL-733A are as follows:

(Mode: FIN)

[2ndF] [CA]
28'000 [+/-] [CFi]

12 [2ndF] [Ni] 316.86 [CFi]

12 [2ndF] [Ni] 335.51 [CFi]

12 [2ndF] [Ni] 353.8 [CFi]

12 [2ndF] [Ni] 371.69 [CFi]

31 [2ndF] [Ni] 389.11 [CFi]

99 [2ndF] [Ni] 389.11 [CFi]

8'389.11 [CFi]

Now, to solve for the blended rate, press:

[IRR] Result: 1.19

Again, because this problem spans 180 periods, many iterations are involved in coming to a solution. This calculation takes a little time. When the result is in the display, annualize this rate by pressing:

[×] 12 [=] Result: 14.27

This 14.27% rate makes sense because the loan starts out at 12%, and goes rapidly to 16%. The majority of the time the loan is at 16%, but the low interest rates at the front of the loan pull the overall rate down, because a significant amount of the balance is paid off at the lower rates.

[n] CALCULATIONS, PARTIAL PERIODS

Calculating [n] has not yet been mentioned in this manual. The reason for not mentioning [n] is that, though on the surface it seems fairly straightforward, when you solve for [n], chances are you will **not** get an integer for an answer. If you solve for [n] and the answer does not come out to be an integer, how are you supposed to interpret that partial period? Look at this example:

Example: You have a credit-line at a local bank. The balance is currently at $3'500. After working through some of the problems in this book and realizing that it is generally much better to be on the positive end of the "interest stick" then on the negative end, you decide it's time to quit drawing on that credit-line and to start making regular payments to get you out of debt for good. The interest rate is 1.3% per month, and you budget $185 per month to pay it off. How long will it take?

Solution: The keystrokes to solve this problem are as follows (make sure that BGN is **not** in the display):

(Mode: FIN)

0 [FV]
1.3 [i]

3'500 [PV]
185 [+/-] [PMT]
[COMP] [n] Result: 21.86

So there you are. It will take 21.86 months to pay off that credit-line. But what does that mean about your last payment? Does it mean that you have to go in exactly 0.86 of the way through the 22nd month and make a full payment? Or does it mean that at the end of the 22nd month you will have to pay 0.86 of a full payment?

All this really tells you is that, in order to pay off that credit-line, you will either have to make a larger payment at the end of the 21st month or a smaller payment at the end of the 22nd month. Remember that explanation back on page 35 that a compound interest loan is like a string of simple interest contracts, each written at the beginning of a period for the length of that period? Unless there is a definition for the way that interest accrues during a period, you cannot use a value for n that is not an integer.

In this problem, since it is unclear as to how interest accumulates during a period, you have to make your final payment either at the end of the 21st month or at the end of the 22nd month. If you key in:

21 [n]
[COMP] [FV],

you will see that the balance of the loan at the end of the 21st month is –156.39. This means that if you want to pay off the loan at the end of the 21st month, you will need to add $156.39 to your regular $185 payment.

If you key in:

22 [n]
[COMP] [FV],

you will see that if you make a full $185 payment at the end of the 22 month, you will have overpaid by $26.58. So your final payment at the end of the 22 month would be 185 – 26.58 = 158.42.

You may also notice that ((158.42 – 156.39) ÷ 156.39) x 100 is 1.30 (to two decimal places). This says that during the 22nd month, the interest that accumulated was exactly 1.3% of 156.39. That conforms to the explanation of compound interest back on page 35.

The previous example points out that in order to calculate correctly using partial periods (where n is not an integer), you have to define ahead of time how interest accumulates during a period.

BOND PRICE

Discussing bond calculations opens up a whole new world of terminology. In this book, we will not even scratch the surface of this new world of terminology, but we will describe a few of the terms and work one common example to demonstrate that, in fact, the cash-flow schedule still applies, regardless of the new words and definitions.

The TVM functions on the EL-733A can handle most bond calculations. Before you read this section you should read the previous section on *Partial Periods*.

Bond calculations take on a slightly different angle than the financial calculations that we have discussed up to now. If you purchase a bond, you know that at some date in the future (called the "maturity date"), that bond will mature and will have a certain value.

The value of the bond upon maturity is called the "redemption value." It is this "redemption value" (a value at the end of the time line) that is usually the starting point for calculations on the bond. This is different from a loan or mortgage where the present value (the amount at the beginning of the time line) is usually the starting point for the calculations. The redemption value of a bond is generally a known.

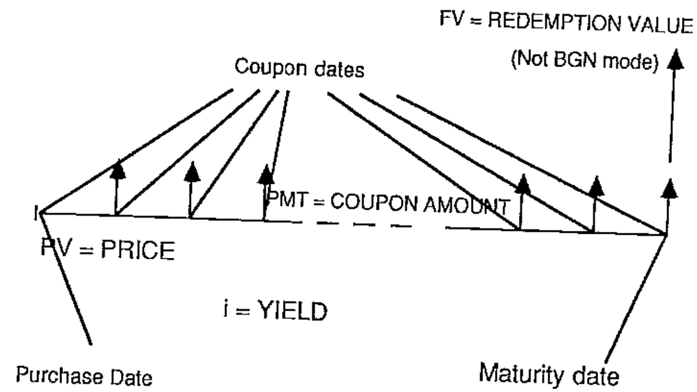
Bonds also have associated with them what is called a "coupon." The "coupon" is a percentage of the redemption value that you will receive **annually** while you own the bond. The "coupon" is like an "interest only payment," but it is based on the future value, the "redemption value" of the bond

On bonds that have "annual coupons," you receive one payment of the coupon amount each year. Some bonds are said to have "semiannual coupons" which simply means

that each year's coupon amount is paid in two equal payments six months apart. The date on which a coupon payment is made is called the "coupon date."

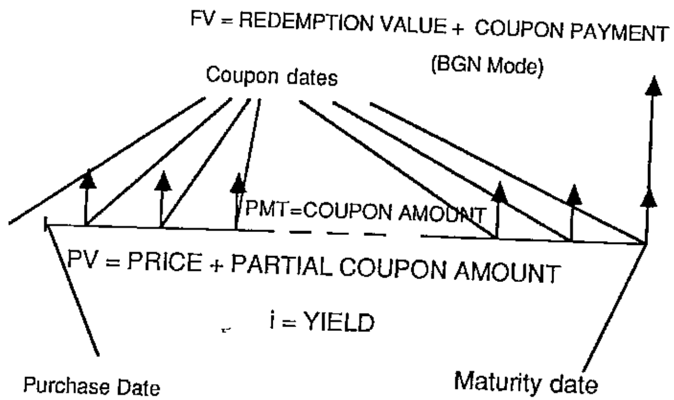
The maturity date is usually the last coupon date and the next to the last coupon date is either 6 months or 1 year before the maturity date. So when you are drawing a cash-flow schedule for a bond, you will usually start at the right end of the time line and work towards the left.

If the day that you purchase a bond falls on a coupon date, these calculations are easy. The previous owner keeps the coupon payment that has accumulated over the past period, and you pay a certain "price" for the bond that depends on what you want to "yield" by owning the bond. The cash-flow schedule for purchasing a bond on one of its coupon dates would look something like this:



The price in this case is just the [PV] of the above schedule and the yield is just [i]. You can solve for either one, given the other. The calculation is performed with the EL-733A out of BGN mode.

However, the purchase date rarely falls on a coupon date, so you almost always have to deal with a partial period. Thus, when you purchase a bond, you have to pay a certain "price" for the bond, plus you have to pay the previous owner the amount of the upcoming coupon payment that has accumulated so far in the current period. So, the cash-flow schedule for a bond usually looks something like this:



The [PV] that you get using the above schedule includes both the price and the amount of the coupon payment that has accumulated in the current period. The yield is still calculated using the [i] function. The calculation requires BGN mode on the EL-733A.

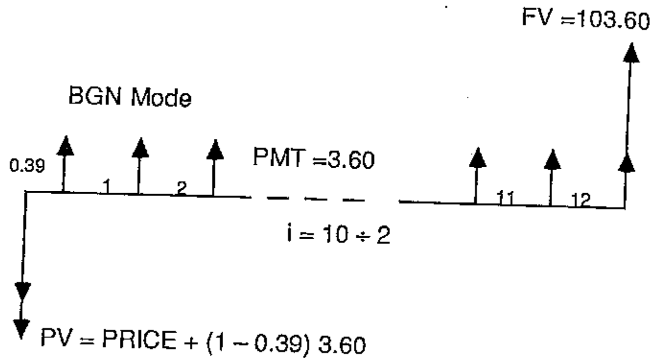
The trick to correctly calculating yield given price or price given yield is in evaluating the length of the partial period at the front of the time line. First you need to count the number of days from the purchase date to the upcoming coupon date. Then, you have to ask the question: how many total days are in the current period? The answer to this question will depend on the calendar used by the bond. Some bonds make it easy by declaring that there are 30 days in every month (or 360 days in a year). Other bonds use the actual calendar, so you have to count the total number of days in the current period. Once you have determined the fraction of the current period that you will hold the bond, you can determine the amount of the upcoming coupon payment that you need to pay to the previous owner in addition to the price.

Example: You are purchasing a 7.2% semiannual coupon bond that is based on 30/360 days. The purchase date is 70 days before the next coupon date, and then it will be six years until the maturity date. Based on a redemption value of $100, what is the price of the bond for which the yield-to-maturity will be 10%?

Solution: A semiannual coupon means that the coupon payment will come every six months. In this case, the coupon payment is (7.2% of 100) ÷ 2 or $3.60. Since the maturity date comes six years after the upcoming coupon date, there is a total of 12 complete periods on the cash-flow schedule.

Because this bond defines one month as 30 days in length, every period is exactly 180 days. Since you know that the purchase date is 70 days before the upcoming coupon date, you know that the partial period on the cash-flow schedule is 70 ÷

180 = 0.39, so n = 12.39. The cash-flow schedule should look something like this:



Because the coupon payments start at the beginning of the first whole period, this is a BGN mode calculation, so the FV contains both the redemption value and the last coupon payment. This boils down to a simple PV calculation where n is not an integer:

(Mode: FIN) BGN mode should be on.

12 [n] 10 [÷] 2 [=] [i]
100 [×] 7.2 [%] [÷] 2 [=] [PMT]
[+] 100 [=] [FV] [COMP] [PV] [+/-] [FV]
0 [PMT] 70 [÷] 180 [=] [n]
[COMP] [PV]

Result: –89.48

You will have to pay $89.48 for this bond to get a 10% yield-to-maturity. But that $89.48 is the price plus the accumulated coupon payment for the current period. For the formal "price" of the bond, subtract that accumulated coupon payment using the following keystrokes:

[x→M] 180 [−] 70 [÷] 180 [×] 3.6 [+] [RM] [=]

Result: –87.28 The price of the bond is $87.28.

If you know the price of a bond and you wish to calculate the yield of the bond, you can get pretty close to an answer using the EL-733A. However, you can't solve for yield directly. The following keystrokes give you an estimate of the yield to maturity if the price of the bond in the above example is $91.33.

3.6 [PMT] 70 [÷] 180 [−] 1 [=] [+/-]
[×] [2ndF] [RCL] [PMT] [+] 91.33 [=] [+/-] [PV]
12 [n] [COMP] [i] Result: 3.83

13 [n] [BGN] [COMP] [i] (End mode) Result: 3.70

The true semi-annual yield lies in between these two rates. You can make guesses at the true yield and then calculate the price as you did in the above example. When the keystrokes result in the correct price, you've arrived at the correct yield. Multiply by 2 to annualize the yield.

Remember, if the purchase date falls on the coupon date, price and yield calculations are simple i and PV calculations and you don't have to deal with partial periods.

DOLLARS AND CENTS -VS- TEN DIGITS
(FINAL PAYMENT CALCULATION)

As a conclusion to this financial calculations chapter, let's look at a subject that you likely will find extraneous, but that is worth mentioning.

Whenever a payment, or any number for that matter, is calculated by the EL-733A, it is computed to an accuracy of 10 digits. However, whenever U.S. money changes hands, it does so to an accuracy of two decimal places (dollars and cents).

On a mortgage payment or any payment stream that extends over a long period of time, if your payment has been calculated to 10 digits, it will be likely be a fraction of a cent high or a fraction of a cent low each time. This small error can accumulate to a few dollars by the end of the contract.

However, as usual, there is a way around this: Whenever you have just calculated a payment using the [PMT] function, key in that payment, rounded to dollars and cents, and then calculate [FV]. By doing this, you accumulate all the fractions-of-a-cent's to a final value which can be incorporated into the last payment:

Example: Calculate the payment on a 30 year mortgage of $95'000 at 10.5% APR (monthly compounding). Accumulate all the inaccuracies into the final payment.

Solution: (Make sure that BGN is not in the display)

95'000 [PV]
0 [FV]
30 [2ndF] [X12] [n]

10.5 [2ndF] [÷12] [i]
(Make sure BGN is off)
[COMP] [PMT] Result: –869.00

That is your payment. It looks like a fairly round number, but if you press [2ndF] [TAB] [•] (to display all the decimal places) you will see that at $869.00 per month, there is a little over 1/5 of a cent that is going unpaid (tsk tsk!). To see what this small error accumulates to over the term of the mortgage, use the following keystokes:

[2ndF] [TAB] [2]
869 [+/-] [PMT]
[COMP] [FV] Result: –5.86

So over 30 years, a $5.86 error has accumulated. Add this value to the regular monthly payment to come up with the final month's payment.

[+] [2ndF] [RCL] [PMT] [=] Result: – 874.86

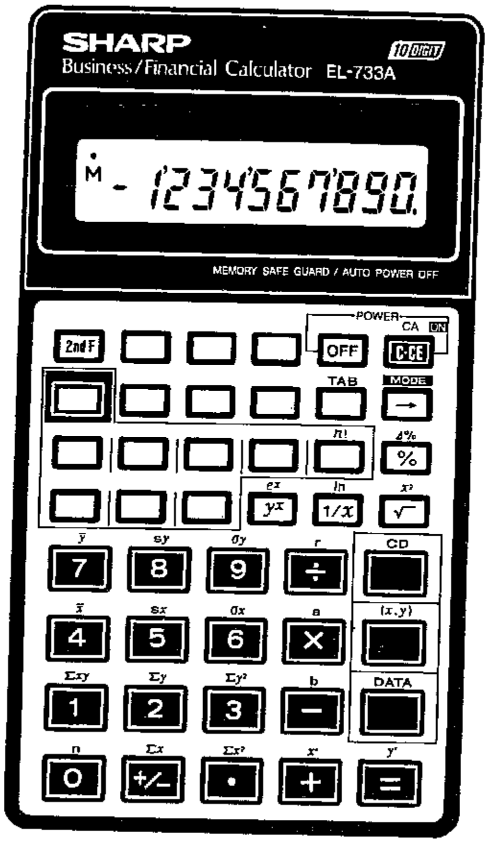
And that is the final payment calculation in this manual.

Chapter 3. Statistics Mode

The EL-733A can be used as a powerful statistics calculator. To activate the statistics functions on the keyboard, you need to put the calculator into STAT mode. Press the [2ndF] [MODE] key until the STAT indicator comes on in the display.

Active Functions In STAT Mode

In STAT mode, the calculator keyboard is as follows:



Only the statistics functions in the bottom four rows of the keyboard are active. Also, the seven math functions and the [%] and [Δ%] functions are active. The memory keys [X→M], [RM], and [M+] change their primary meanings to become the [CD], [(x,y)], and [DATA] keys. You will use these three keys to accumulate the numbers (data) that you use in your statistical calculations. The STAT mode meanings of these three keys are as follows:

[DATA] Accumulate data.

[(x,y)] Two variable statistics, data accumulation.

[CD] Correct a data entry.

The EL-733A offers functions for both one-variable and two-variable statistical calculations. One-variable statistics is used for totaling a list of numbers and for calculating the mean ([$\bar{x}$]), standard deviation ([Sx] or [σx]), and sum of the squares ([Σx^2]) of a list of numbers. Two-variable statistics can be used to perform all the functions of one-variable statistics on a list of number pairs, plus it can be used for linear regression (which is mathematically approximating a straight line through a set of "x,y" data pairs), for linear forcasting (forcasting on a straight line), and for exponential regression and forcasting.

Most of the explanation in this chapter on statistics assumes that you have some knowledge of the statistical functions you want to use. You should know the definitions of terms like "mean," "population standard deviation," and "sample standard deviation." That is, you should know that the *mean* is the average of a set of numbers and that *standard deviation* is basically a measure of the fluctuation of the individual data points around the mean.

If you are going to use linear regression, you should know the formula for a straight line on a two dimensional (x,y) coordinate system (that formula is y = ax+b where a is the "slope" of the line and b is the "y intercept") and you should have at least brushed shoulders with the term "correlation" or "correlation coefficient." The function [2ndF] [r] calculates the correlation coefficient for a set of data pairs in a linear regression problem. The correlation coefficient is a measure of how close the data points in the set fall to the straight line that approximates their path.

However, you may be able to get all the information you need out of the brief explanation and examples that are given in this chapter. So give it a try, even if you never have heard any of the above statistical terms.

In any statistical calculation, once you have your numbers keyed in correctly, the work is done. As long as you feed them the correct numbers, the statistical functions on the EL-733A take care of all the involved number crunching. In the following section of this chapter, we look at how you key in a list of single values and at the one-variable statistics functions that you can apply to that list of single values.

Single Variable Statistics

In the example back on page 24, you were the owner of a trucking company that reduced its fuel costs by installing wind deflectors on the cabs of all the tractors in the fleet. One of the numbers that you worked with in that problem was the "average" mileage. But where does that number come from? What is the easiest way to calculate an average mileage from a fleet of, say, twelve trucks?

KEYING IN DATA AND THE [2ndF] [x̄] FUNCTION

Assume that you are given a stack of mileage reports at the end of a month. The twelve drivers in your company have

all calculated miles-per-gallon, based on the miles they put on their trucks and the amount of fuel their trucks consumed during the month. The mileage figures are as follows:

Truck Number	Mileage
1	7.13
2	4.97
3	6.26
4	7.34
5	5.69
6	6.95
7	4.03
8	6.57
9	5.85
10	7.42
11	6.11
12	4.67

To calculate the average or "mean" for the above mileages, just key in each number and press [DATA]. ([DATA] is the new meaning of the [M+] key in STAT mode. If you are not in STAT mode, press the [2ndF] [MODE] key until the STAT indicator comes on in the display.) Once you have all the numbers keyed in, press [2ndF] [x̄] to calculate the mean.

Now key in that list of mileages and calculate the mean. The keystrokes are as follows:

(Mode: STAT)

[2ndF] [CA]
7.13 [DATA]
4.97 [DATA]
6.26 [DATA]
7.34 [DATA]
5.69 [DATA]

6.95 [DATA]
4.03 [DATA]
6.57 [DATA]
5.85 [DATA]
7.42 [DATA]
6.11 [DATA]
4.67 [DATA]
[2ndF] [$\bar{x}$] Result: 6.08

Your fleet of trucks averaged 6.08 miles per gallon during this last month.

THE [DATA] KEY AND THE [2ndF] [n] FUNCTION

Pressing the [DATA] key in STAT mode after entering a single datum accumulates that datum for the statistical functions. When you press this key, you will notice that the calculator keeps track of how many data you have keyed in by counting them in the display. This displayed number is called "n." Pressing [2ndF] [n] brings "n" to the display.

In one-variable statistics, the data are referred to in the function names as "x." Thus, for one-variable statistics, the functions that you can use are [$\bar{x}$] (x-bar or mean), [Sx] (sample standard deviation), [σx] (population standard deviation), [Σx] (the sum of the x's), and [Σx^2] (the sum of the x's after they have been squared).

STANDARD DEVIATION

To calculate the standard deviation for the above list of mileages, you first have to determine which type of standard deviation applies. The [2ndF] [Sx] key calculates the "sample standard deviation" and the [2ndF] [σx] key calculates the "population standard deviation."

The "sample standard deviation" assumes that the data you have keyed in are **a sample of a large population** for which you are trying to estimate the standard deviation.

The "population standard deviation" assumes that the data you have keyed in are **the entire population**. In the case of the truck mileages, the "population standard deviation" is the one you would want to calculate, because you own only 12 trucks and those 12 trucks are all you are interested in (they are the entire population). If your calculations were, for example, for the benefit of the EPA to estimate the mileage for a certain model truck based on a sample of 12 road tests, the "sample standard deviation" would be your deviation of choice, because the EPA would take your results and apply them to all trucks of that model.

For the list of numbers that you have keyed in, press [2ndF] [σx] to calculate the population standard deviation (1.05) and press [2ndF] [Sx] to calculate the sample standard deviation (1.09). The **variance** is the standard deviation squared. Press [2ndF] [x^2] to calculate variance after calculating the standard deviation.

THE [Σx] AND [Σx^2] FUNCTIONS

As mentioned before, the [Σx] function returns the total of the list of numbers that you enter using the [DATA] key, and the [Σx^2] function returns the result that you would get if you squared all those numbers and then added them together.

In the average mileage example that we have used in this section, the total and the sum of the squares would probably not be something you were interested in knowing (unless you were going to use those values in some other statistical formulas). However, if you were interested in those values, you would find that pressing [2ndF] [Σx] would give you a result of 72.99 and that pressing [2ndF] [Σx^2] would give you a result of 457.11.

CORRECTING A DATA ENTRY

Occasionally we humans make errors, especially when it comes to keying in long lists of numbers. Fortunately, the EL-733A comes with a function that allows us to correct any errors we make when keying in statistical data. In STAT mode, the [x·M] key changes its primary meaning to [CD] which stands for "correct data."

As a demonstration of correcting a data entry, refer to the example "average mileage" calculation back on page 148 and assume that after you had completed your calculation of the mean mileage, the driver of truck number 4 informed you that he had made an error in *his* calculation (he didn't have a SHARP calculator). After checking his numbers he found that his mileage for the month was actually 7.20, not 7.34.

To correct that error and recalculate the mean based on the accurate data, the keystrokes are as follows:

7.34 [CD]
7.20 [DATA]
[2ndF] [$\bar{x}$] Result: 6.07

Notice as you are making the above changes that when you press 7.34 [CD] the calculator displays 11.00 indicating that it has removed one of the 12 data. When you add the correct datum by pressing 7.20 [DATA], the calculator displays 12.00, which is the number of data in the complete list.

Two-Variable Statistics

When you add a second dimension (a second variable) to statistical calculations, the number of potential applications increases considerably. Again, the most time-consuming part of solving a two-variable statistics problem on your EL-733A is just keying in the (x,y) data pairs.

The statistical functions on the EL-733A that require the input of two-variable data are as follows:

[2ndF] [x'] Return a predicted x for a given y.

[2ndF] [y'] Return a predicted y for a given x.

[2ndF] [$\bar{y}$] Mean of the y's.

[2ndF] [Sy] Sample standard deviation of y's.

[2ndF] [σy] Population standard deviation of the y's.

[2ndF] [Σxy] Sum of the individual pair products: x times y.

[2ndF] [Σy] Sum of the y's.

[2ndF] [Σy²] Sum of the y's after they have been squared.

[2ndF] [r] Correlation coefficient.

[2ndF] [a] y-intercept in the regression formula y = a + bx.

[2ndF] [b] Slope in the linear regression formula y = a + bx.

KEYING IN DATA

The two numbers in the statistical data pairs that you use in two-variable statistics are called "x" and "y." To key in a data pair, key in x, press the [(x,y)] key (which is the STAT mode meaning of the [RM] key), key in y, and press [DATA]. To correct a data entry, key in the incorrect x, press the [(x,y)] key, key in the incorrect y, and press [CD].

As an example, assume you invested in a company in January of 1982. The value of the stock has increased in value steadily according to the following table.

January of:	$ Stock Value
1982	45'000
1983	46'976
1984	49'254
1985	51'770
1986	52'624
1987	54'190

What has been the average value of your investment during those years? And, a more interesting question is: if the rate of increase in the stock value can be approximated by a straight line, what will your stock be worth in the year 2000?

You have six data pairs to key in for this problem. They are (1982, 45'000), (1983, 46'976), (1984, 49'254), (1985, 51'770), (1986, 52'624), (1987, 54'190). The keystrokes used to key in this data are as follows:

(Mode: STAT)

[2ndF] [CA]
1982 [(x,y)] 45'000 [DATA]
1983 [(x,y)] 46'976 [DATA]

1984 [(x,y)] 49'254 [DATA]
1985 [(x,y)] 51'770 [DATA]
1986 [(x,y)] 52'624 [DATA]
1987 [(x,y)] 54'190 [DATA]

Remember that [(x,y)] is the STAT mode meaning of the [RM] key. [(x,y)] is not the second function of the [RM] key, so you do not have to press [2ndF] first (you can if you wish though).

As you key in the above six data pairs, you will notice that the calculator keeps track of the pair that you are on by counting them in the display. This number that it shows in the display is called "n." At any time, you can see how many data pairs have been keyed in by pressing [2ndF] [n]. At the end of keying in the data, n is equal to 6.00, and 6.00 is what should be showing in the display.

Once you have the data keyed in, you are first after the answer to the question: what is the average value of your stock over the years? To see this answer press:

[2ndF] [$\bar{y}$] Result: 49'969.00

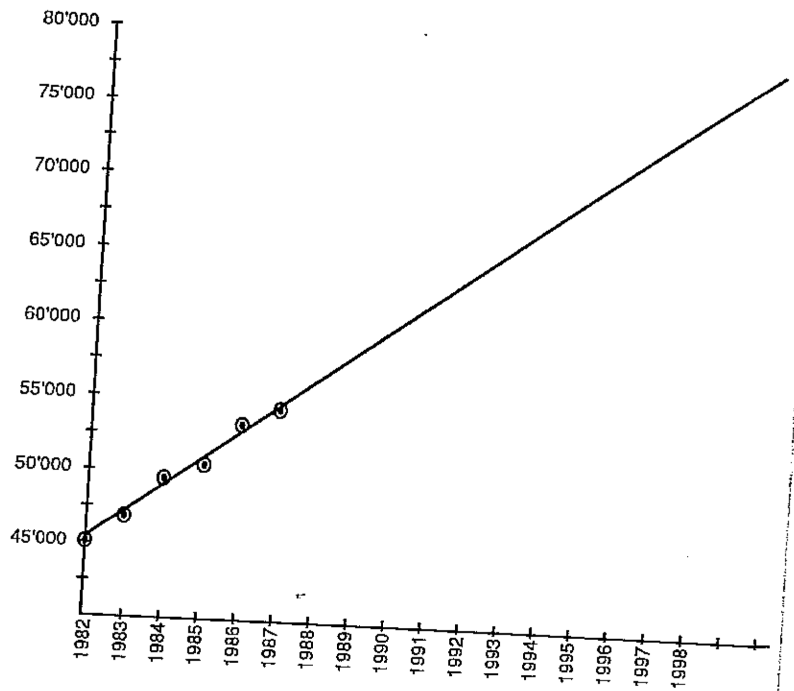
The [$\bar{y}$] key returns the mean of the y values. In this case, you entered the stock value as the y component of each data pair, so [$\bar{y}$] returns the average stock value over the years.

The next question you want to answer is: if the increase in value can be approximated by a straight line (in other words, if the trend of the last five years stays constant) what will the stock value be in January of the year 2000?

To answer this question, simply use the keystrokes:

2000 [2ndF] [y'] Result: 78'936.29

This graph shows the six data points plotted with a line drawn to approximate their trend over the years 1982 to 1987. The line is extended to the year 2000 where it predicts the stock value to be at $78'936.29:



Linear Regression Of Stock Value Data

Of course, this is just a simple example to demonstrate the [y'] function. It would probably not be wise to use a linear extrapolation on a five-year trend in stock value to evaluate the distant future of an investment.

SLOPE, Y-INTERCEPT, AND CORRELATION

Three important values in linear regression are [a], [b], and [r]. The values [a] and [b] come from the equation for a line (y = a + bx), where [a] is the point that the line crosses the y-axis (the vertical axis) and [b] is the slope of the line. With the equation for a line, you can describe any straight line. The value [r] is called the "correlation coefficient," and it is a measure of how closely the data points fit the line described by [a] and [b].

The correlation coefficient [r] ranges from –1 to 1. The closer this value is to 1 or –1, the closer the data points are to the line. For a certain set of data, if [r] is close to zero, the linear correlation is poor, which means that a straight line is a poor choice for modeling that set of data.

To calculate [a], [b], and [r] for the previous example, use the following keystrokes:

(Mode: STAT)

[2ndF] [TAB] [•] (to see all the decimal places)

[2ndF] [a] Result: –3'658'778.

[2ndF] [b] Result: 1868.857143

[2ndF] [r] Result: 0.990733242

The reason that [a] is such a large negative number in this case, is that the line stretches all the way back to the year 0000 before it crosses the y-axis.

So to fit a set of (x,y) data pairs to the curve $y = ab^x$, you simply enter your pairs as usual except, after you enter y, press the [2ndF] [ln] keys. Also, whenever you calculate [y'], [a], or [b], press the [2ndF] [e^x] keys (because $e^{\ln(y)} = y$).

Example: Fit the following data to a curve described by the equation $y = ab^x$. Determine the values for "r," "a," and "b," and calculate the predicted y values at x = 9.2 and x = –2.6.

x	y
0.5	8.0
1.6	13.2
3.6	52.9
7.9	1008.0
8.7	2201.0

Solution: To enter the data, use the following keystrokes:

(Mode: STAT)

[2ndF] [CA]

.5 [(x,y)] 8 [2ndF] [ln] [DATA]
1.6 [(x,y)] 13.2 [2ndF] [ln] [DATA]
3.6 [(x,y)] 52.9 [2ndF] [ln] [DATA]
7.9 [(x,y)] 1'008 [2ndF] [ln] [DATA]
8.7 [(x,y)] 2'201 [2ndF] [ln] [DATA]

Once the five data points are entered, you can calculate the desired regression values using the keystrokes below:

[2ndF] [r]	Result: 0.998657732
[2ndF] [a] [2ndF] [e^x]	Result: 4.868338038
[2ndF] [b] [2ndF] [e^x]	Result: 1.988330772

The correlation coefficient is 0.998... indicating that this curve is an excellent model for this data. Also from the above results, the equation for the curve is:
$y = 4.87\ (1.99^x)$

To calculate the predicted values for y at x = 9.2 and x = –2.6, press:

9.2 [2ndF] [y'] [2ndF] [e^x]	Result: 2'713.164694
2.6 [+/-] [2ndF] [y'] [2ndF] [e^x]	Result: 0.815286551

Appendix A. Care And Maintenance

Since the liquid crystal display is made of glass material, treat the calculator with care. To insure trouble-free operation of your SHARP calculator, we recommend the following:

1. The calculator should be kept in areas free from extreme temperature changes, moisture and dust. During warm weather, vehicles left in direct sunlight are subject to high temperature build-up. Prolonged exposure to high temperature may damage your calculator.
2. A soft, dry cloth should be used to clean the calculator. Do not use solvents or a wet cloth.
3. If the calculator will not be used for an extended period of time, remove the batteries to avoid possible damage caused by battery leakage.
4. If service of your calculator is required, use only an authorized SHARP SERVICE CENTER listed on the last page of this manual.
5. Keep this manual for reference.

How To Proceed If Abnormal Conditions Occur

If this calculator is exposed to a powerful external electric field or shock during use, a rare condition may occur in which all the keys, including the [ON/C] key, do not function. In this case, press the **RESET** switch on the back of the calculator. Note that the memory contents will be completely cleared when this operation is performed.

Press the **RESET** switch only in the following cases:

- After replacing the batteries
- To clear all memory contents

- When an abnormal condition occurs and all keys are inoperative.

Battery Replacement

Batteries Used
Alkaline-manganese battery (LR44) x 2

Notes on Battery Replacement
Batteries that are not properly handled can leak and damage the calculator. For this reason, pay particular attention to the following points.

- Be sure to turn the power off when replacing batteries.
- Replace both batteries at the same time.
- Do not mix new and used batteries.
- Use the same type of batteries.
- When installing, orient each battery properly according to the + markings as indicated in the calculator.
- **Keep the batteries out of the reach of children.**
- Since the original batteries were factory-installed before shipment, they may require replacement before the continuous operating time indicated in the specifications is reached.
- Exhausted batteries left in the calculator may leak and damage the calculator.
- Do not throw the batteries into a fire, as they may explode.

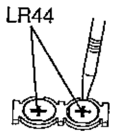
When to Replace Batteries
When the battery indicator is out, replace the batteries. Immediately change them by following the procedure below.

Replacement Procedure

1. Turn the power off by pressing the [OFF] key.
2. Remove the two screws on the back and remove the back cover.

3. Do not touch any of the internal electronic components. Remove the used batteries by prying them with a ball-point pen or other similar pointed device as shown in the figure.
4. Install two new batteries (alkaline-manganese, LR44 or equivalent) with their positive sides facing up.
5. Replace the back cover and secure it with the screws.
6. Press the RESET switch, located on the back of the calculator.

(Figure)



Make sure that the display appears as shown below display and then press CA C·CE . If the display does not appear as shown, remove the batteries, reinstall them and check the display once again.

Appendix B: Specifications

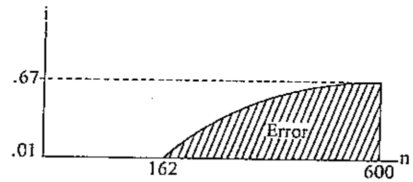
Model:	EL-733A
Display capacity:	Ten digit floating decimal point display or an eight digit mantissa and two digit exponent. Minus symbol appears both in mantissa and exponents portion.
Number of internal calculation digits:	Mantissa: 12 digits, Exponent: 2 digits.
Calculations:	Four arithmetic calculations, reciprocal, square root, square, and power, logarithm and exponential, factorial, memory, statistical calculations, and financial calculations.
Display:	Liquid Crystal Display (FEM type)
Component:	LSI etc.
Power supply:	3.0V DC Alkaline manganese battery (LR-44) x 2
Power consumption:	3.0 V DC (0.00025W)
Operating time:	Approx 1000 hours (LR-44) of displaying 55'555 at ambient temperature: 20° C (68°F). The operating time will vary depending on the type of battery and the way the calculator is used.
Operating temp.:	0°C to 40°C (32°F to 104°F)
Dimensions:	76(W) x 143 (D) x 8.5 (H) mm 3"(W) x 5-5/16"(D) x 11/32"
Weight:	Approx 110g (0.24 lb.) (including wallet)
Accessories:	Alkaline manganese battery x 2 (built-in) and owner's manual

– Range Limitation –

There are two processes that can take place in finance; amortization (systematic elimination of debt) and growth (systematic accumulation of wealth).

When solving for interest rate (i) in a growth problem, the calculator may display the error symbol "E" if the value for the number of payments is between 162 and 600 and the anticipated value for interest rate (i) falls between .01% and .67% per compounding period.

The graph and cash flow sign conventions below highlight the area of concern.



When:

{ PV≤0, PMT<0, FV>0 } or { PV≥0, PMT>0, FV<0 }

Error Conditions

In case of error, the symbol "E" will be displayed. An error will be caused by calculations or instructions beyond the capacity of the machine, as listed below. An error can be cleared by the [C·CE] key.

1. When the absolute value of a calculation result is greater than 9.999999999 x 10^{99}.
2. When a number is divided by zero (A ÷ 0).
3. For special functions, an error occurs when the calculations exceed the ranges specified in the previous tables.

Appendix C. Indexes

Function Index